Using Storage to Minimize Carbon Footprint of Diesel Generators for Unreliable Grids

Sahil Singla*, Yashar Ghiassi-Farrokhfal†, and Srinivasan Keshav‡

Abstract—**Although modern society is critically reliant on power grids, even modern power grids are subject to unavoidable outages. The situation in developing countries is even worse, with frequent load shedding lasting several hours a day due to a large power supply- demand gap. A common solution for residences is, therefore, to backup grid power with local generation from a diesel generator (genset).**

To reduce carbon emissions, a hybrid battery-genset is preferable to a genset-only system. Designing such a hybrid system is complicated by the tradeoff between cost and carbon emission. Towards the analysis of such a hybrid system, we first compute the minimum battery size required for eliminating the use of a genset, while guaranteeing a target *loss of power* probability for an unreliable grid. We then compute the minimum required battery for a given genset and a target allowable carbon footprint.

Drawing on recent results, we model both problems as buffer sizing problems that can be addressed using stochastic network calculus. Specifically, a numerical study shows that, for a neighbourhood of 100 homes, we are able to estimate the storage required for both the problems with a fairly small margin of error compared to the empirically computed optimal value.

Index Terms—**Smart grids, Batteries, Diesel engines, Performance analysis, Power system reliability, Power demand.**

I. Introduction

The power grid underlies most modern societies: power failures can affect critical institutions such as hospitals, aircraft control towers, and Internet data centres. Despite this great reliance on electrical power, the situation in many developing countries is marginal, with daily load shedding lasting two-to-four hours due to demand spikes and unreliable generation [1].

In the face of this inherent unreliability, a common solution is for critical facilities, and even some individual homes, to augment grid power with local generation, typically from a diesel generator (genset). This, however, increases the carbon footprint of the load [2]. One of the best solutions to this problem is using a hybrid system that combines gensets with storage batteries (along with two-way inverters to convert between AC and DC power) [3], [4]. We study the use of such a hybrid system to allow a set of homes in a single residential neighbourhood to avoid power outages. Because storage battery is expensive, the design of a hybrid system requires a trade-off between carbon emission and the cost of storage.

The design of such a hybrid system for unreliable grids is a technically complex problem. In this work, we analytically approach this problem for the first time. We first consider the case where reducing residential neighbourhood carbon emission to zero is desirable, and we size a battery for a given maximum loss of power probability in the absence of a genset. Thereafter, we consider the case where a genset is available, and we compute the minimum battery size for a given maximum allowable genset carbon emission. We note that these two problems are technically different as they have different constraints; loss of power probability is the constraint in the absence of genset and total carbon footprint is the constraint in the presence of genset (the presence of genset ensures that the demand is always met).

The complexity of the problem comes from choosing an appropriate model for the stochastic nature of household loads [5]. Indeed, it has been shown to be isomorphic to the complex (but well-known) problem of choosing a buffer large enough to smooth the data being generated by a variable-bit-rate traffic source [6], [7]. Therefore, drawing on recent results [7], we approach the solutions to our problem using the powerful techniques of stochastic network calculus.

We have numerically evaluated the accuracy of our algorithms using real traces of electrical loads collected over 12 months from 4500 homes in Ireland [8]. Through numerical examples we show that given the target power outage probability is upper bounded by one day in ten years (a standard risk target in power engineering), our approach computes a battery size that is only about 10% more than the minimum battery required had the future load been exactly known. Moreover, we are able to estimate the carbon footprint reduction, compared to exactly known future load, within a factor of 1.7.

The key contributions of our work are:

1) We use a stochastic network calculus approach to characterize the stochastic electrical load and find the genset carbon emission as a function of the battery size, thus allowing us to estimate the battery size needed to limit carbon emissions of a residential neighbourhood.
2) Given a load characterization, we analytically compute the smallest battery size necessary to eliminate the use of genset and meet a target loss of power probability for a load connected to a highly *unreliable* power grid.
3) We use a public dataset of electricity consumption data from 4500 homes to compare our carbon emission and battery size bounds to those computed empirically and from bounds obtained using teletraffic theory. We find

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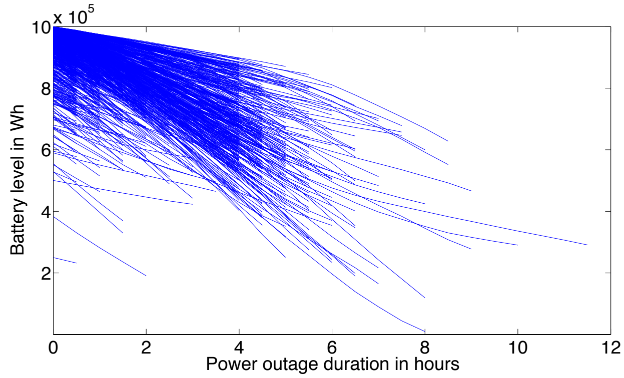


Fig. 1: Trajectories of battery level during load shedding periods for $B = 1MWh$ and $C = 10^5W$.

that our bounds are quite close to the empirical optimal and far better than the prior approaches.

The rest of the paper is laid out as follows. Section II starts with a motivation for the problem along with the prior work. Section III presents the necessary background in teletraffic networks and stochastic network calculus, and discusses the prior known bounds. The model used to analyze battery sizing problem along with our assumptions is explained in Section IV. The bounds on minimum battery size, both in presence and absence of genset, are computed in Section V. We show the tightness of our bounds numerically on a public dataset in Section VI. Finally, Section VII concludes the paper with the limitations and future work.

II. Motivation and Related Work

We define *loss of power* as the state of battery depletion. In this state in the absence of a genset, demand cannot be met by the system, and in presence of a genset, causes carbon emission from the genset to meet the demand. Figure 1 illustrates examples of the trajectories of battery level (also called the battery state-of-charge) of different lengths for real measured loads from a public electricity demand dataset (details in Section VI). In this numerical evaluation, the battery is fully depleted only once. Note that had the battery size been smaller, say 400 KWh, we would have had a much larger number of loss of power events. The figure shows how unpredictable the battery level can be due to the randomness of the demand energy and the size of load shedding interval.

It is clear that the probability of a loss of power depends on many factors, including the battery size, its charging rate limit, load during a loss of power event, the distribution of the inter-load-shedding intervals, and the distribution of these durations. We seek to compute the tail probability of the state-of-charge distribution of the battery; an inherently complex problem. However, it has been recently shown that this storage sizing problem is very similar to the buffer-sizing problem in a telecommunications network. We are inspired by these theoretical results in [7], [9], which adopt a queuing-theoretic buffer-sizing analysis to size batteries, to use a similar approach based on stochastic network calculus, to study the problem of battery sizing to limit genset carbon emission for unreliable grids.

Prior work in this area relies primarily on empirical numerical analysis rather than analytical modeling [3], [10], [11]. Carbon emission due to a battery-genset hybrid system has been studied mainly through the notion of genset *efficiency*, which is the diesel consumption per unit energy production [3], [12]. We note that some of the prior analytical work (e.g. [9]) assumes load stationarity. In contrast, our approach does not explicitly assume stationarity.

A probabilistic loss of power formulation is presented in [7] for an intermittent power resource (e.g., wind or solar power) serving a stochastic demand with the aid of batteries. The framework in [7] considers a battery-only system, and genset existence or the trade-off between storage size and genset carbon emission is not studied. Moreover, even for the battery-only system, that framework cannot be directly used for an unreliable system due to a major system model difference; in an unreliable gird, if the grid is available then it is assumed to be large enough to serve both the instantaneous demand and charge the battery at its maximum charging rate C. This is not the case for renewable energy sources as they can be available in a time slot yet not large enough to meet the demand and charge the battery. In other words, in an unreliable grid scenario, the power source at any time is either infinite (if available) or zero (if unavailable), whereas renewable energy sources have limited power at any given time. This makes the power source modelling presented in [7] inapplicable to model the power source in an unreliable grid scenario.

III. Background and Preliminaries

The problem of battery sizing in power distribution systems can be mapped to the problem of buffer sizing in the teletraffic network. This analogy has been used in some recent papers, borrowing state-of-the-art analytical results from teletraffic theory [6], [7], [13]. We briefly discuss the essence of this analogy in this section.

A. Deterministic loss of power vs. deterministic loss of traffic

Suppose that an arrival process A enters a buffer (queue) of size B, which can serve traffic at rate C, and let A' be the corresponding departure process. We assume a discrete time model with time unit t_u, where events can only happen at discrete time instants, i.e. at $t = 0, t_u, 2t_u, 3t_u, \ldots$ To simplify notation, we assume $t_u = 1$ for the rest of the paper. Thus, we frequently write C to mean Ct_u in the following equations. The distinction will be clear from the dimensions whether C means the service rate or the amount of service in a single time unit. We denote the total arrival from process A in time interval $[0, t]$ by $A(t)$ and we use $A(s, t)$ to mean $A(t) - A(s)$.The *backlog* $b(t)$ at any time t is defined to be the buffer content at that time and is given by the following recursive equation:

$$b(t) = \min(B, [b(t-1) + A(t-1, t) - C]_+) \ , \qquad (1)$$

where $[x]_+ = \max(0, x)$ for any value of x. Eq. (1) is equivalent to the following non-recursive expression [14]:

$$b(t) = \min_{0 \le u \le t} \Big(\max_{u \le s \le t} \Big(A(s,t) - C.(t-s),$$

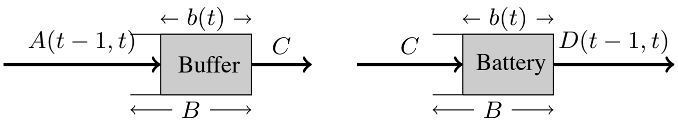


Fig. 2: Loss of traffic due to buffer overflow vs. loss of power when a demand finds the battery empty.

$$A(u,t) - C.(t-u) + B\Big)\Big) \ . \quad (2)$$

Given the buffer content at time $t-1$, the *loss of traffic* due to buffer overflow at any time t is

$$l(t) = [A(t-1,t) - C + b(t-1) - B]_+ \ . \quad (3)$$

Eqs. (2-3) can be combined to extract the following loss characterization [14]:

$$l(t) = \min_{0\le u\le t-1}\Big(\max_{u\le s\le t-1}\Big([A(s,t) - C.(t-s) - k(t)]_+, \\ [A(u,t) - C.(t-u) + k(u) - k(t)]_+\Big)\Big) \ , \quad (4)$$

where

$$k(t) = \begin{cases} B & t > 0 \\ 0 & t = 0 \end{cases} \quad (5)$$

There is an analogous problem in the power system as follows. A constant power source with rate C is feeding a battery with size B, and the battery is used to serve an intermittent demand D (see Figure 2). The *deficit battery charge* b^d is defined as the amount of energy needed to fully charge the battery and is given by

$$b^d(t) = \min(B, [b^d(t-1) + D(t-1,t) - C]_+) \ , \quad (6)$$

where $D(t-1,t)$ is the demand at time slot t. We note that the above equation is dimensionally consistent, because we assume discrete time unit $t_u = 1$, and technically C represents Ct_u here. A comparison between Eq. (1) and Eq. (6) suggests that the deficit battery charge is mapped to the backlog status, if the demand D is mapped to the arrival traffic A, the power supply is mapped to the capacity (service rate) of the link, and the battery size is mapped to the buffer size [6].

The *loss of power* is defined as the event that a demand finds the battery empty or, equivalently, finds the deficit charge of the battery full. The loss of power at time t is given by $[b^d(t-1) - C + D(t-1,t) - B]_+$, which is comparable to Eq. (3). Thus, the non-recursive equations for the backlog and the traffic loss in a finite buffer system in Eqs. (2) and (4) can, respectively, be used to compute the deficit state of the charge and the loss of power in energy systems [6], [15].

B. Probabilistic loss formulations

As Eq. (4) suggests, the loss analysis requires an accurate characterization of the statistical properties of the traffic arrival (demand power). Extensive results exist from the teletraffic theory, using the method of large deviations, when the number of independent arrivals is large (*many sources asymptotic regime*) [16]. Unfortunately, this approach cannot be applied to the energy systems as there are not *many independent* demands sharing the same battery in a distribution energy system.

One of the (arguably) most suitable alternative analytical approaches for the energy systems is the theory of *Network Calculus* [17], [18], [19]. This theory uses envelopes to characterize stochastic processes and to compute performance bounds.

Several probabilistic envelopes have been proposed in the literature (see [20] for a review). Here we use a concept called the *statistical sample path envelope* [21]. A non-decreasing function $\mathcal{G}$ is a statistical sample path envelope for an arrival process A with bounding function ε if it satisfies the following at any time $t \ge 0$ and for any $\sigma \ge 0$

$$\Pr\left\{\max_{s\le t}\{A(s,t) - \mathcal{G}(t-s)\} > \sigma\right\} \le \varepsilon(\sigma) \ , \quad (7)$$

where $\varepsilon(\sigma)$ is non-increasing in σ.

Having a statistical sample path envelope on the demand power A, an upper bound on the loss probability can be achieved from Eq. (4). Due to the complexity of Eq. (4), the following upper bound is used in [14] to extend the traffic loss formulations to the probabilistic settings:

$$l(t) \le \max_{0\le s\le t}([A(s,t) - C.(t-s) - B]_+) \ . \quad (8)$$

The traffic loss upper bound in Eq. (8) (from teletraffic theory) along with the underlying mapping between the loss of power and the traffic loss are used in [7] to obtain an upper bound on the loss of power probability:

Theorem 1 (Loss of power probability [7])**.** *Suppose that $\mathcal{G}$ is a statistical sample path envelope for a demand process D in the sense of Eq. (7) with bounding functions ε_g. If this demand is served by a constant power source C fed to a battery of size B, the loss of power probability satisfies the following:*

$$\Pr\{l(t) > 0\} \le \varepsilon_g\left(B - \max_{0\le\tau\le t}(\mathcal{G}(\tau) - C\tau)\right) \ . \quad (9)$$

If a measurement trace of a process A is given, the statistical sample path envelope $\mathcal{G}$ can be computed by (a) constructing a set Y consisting of the sample values for the event $\max_{s\le t}\{A(s,t) - \mathcal{G}(t-s)\}$ for each trajectory and at any time t and (b) using the complementary cumulative distribution function (CCDF) of the sample set Y as a bounding function $\varepsilon(\sigma)$ for Eq. (7) [22].

IV. System Model

The system model considered in this paper is illustrated in Figure 3. The grid and the battery are used to serve 'most' of the demand. The grid is available irregularly. If the grid is available, then it is used to serve the demand and charge the battery. When the grid is not available (power outage), the charge of the battery is used to serve the demand. In the presence of a genset (not shown in Figure 3), the genset starts running only when the battery is empty during a load shedding.

Denote by $d(t)$ the energy demand at time slot t, and by

$D(t)$ the cumulative energy demand in time interval $[0, t]$[1]. To simplify notation, we define $D(s, t) = D(t) - D(s)$. The charging rate of the battery is represented by C and the battery size by B. Let $x(t)$ be a binary random variable representing the availability of the utility grid at time slot t, *i.e.*, $x(t) = 0$ if grid is unavailable (power outage) at slot t, and $x(t) = 1$, otherwise. We define $x^c(t) = 1 - x(t)$ as the complement of $x(t)$. If the grid is available at time slot t (i.e., $x(t) = 1$), the energy demand is served by the grid and the battery will be charged by as much as C energy unit in that time slot. On the other hand, if the grid is not available (i.e., $x(t) = 0$), the energy demand must be served by the energy stored in the battery.

We follow two objective functions for battery sizing in this paper. First, in the absence of genset, we size the battery B[2] such that given some statistical properties of the energy demand process, the probability of loss of power is kept below a target threshold ϵ^*. Second, in the presence of genset, we size the battery such that the total carbon footprint is kept below a certain threshold.

We have the following assumptions in our formulations

1) Battery capacity does not fade during its lifetime or overall time duration we are studying it. This means that the battery state of health (SOH) is always 100%.
2) Battery charging rate is upper bounded by C, but there is no constraint on the discharging rate. [3]
3) Genset is large enough to meet the maximum loads.

In practice, SOH is a large value throughout the lifetime of a battery and if it decays below a large threshold, the battery is assumed dead and non-functional. Thus, one can assume that changes in battery capacity over its lifetime are small and can be ignored. The other assumptions typically hold in practice since battery charging rate depends on the technology of the battery, and for technologies such as lead-acid battery, the discharging rate is multiple times higher than the charging rate [23]. Moreover, the marginal cost of increasing genset size is negligible compared to the marginal cost of a battery [24], [25].

V. Battery Sizing Formulation

The unreliable grid described in our problem statement can be converted to a reliable *compound* power source i.e., one that is augmented by batteries. In this section, we compute the required battery size for 1) a target loss of power probability in the absence of a genset, and 2) for a target carbon emission threshold in the presence of a genset. The size of the battery is a function of the stochastic nature of demand and grid unavailability.

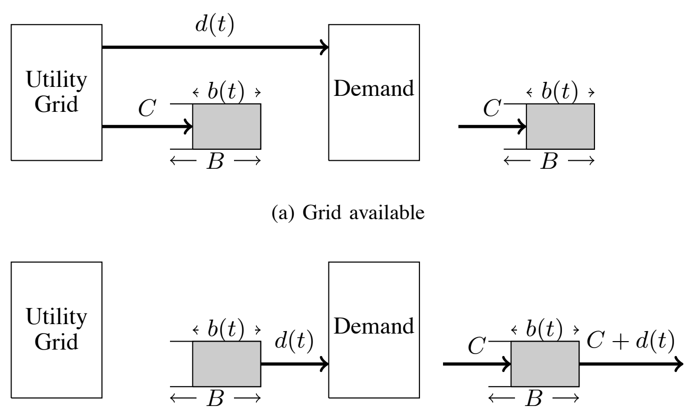
[1]The energy demand varies widely over the course of a year showing marked seasonality. Our analysis is agnostic to the time interval over which the demand is modeled. In practice, however, similar to the concept of busy-hour sizing in a telecommunication network, we advocate the sizing of a battery keeping in mind the underlying non-stationarity of the demand process [9].

[2]Note that we seek to minimally size the *effective* battery storage capacity. Thus, if the maximum allowable depth of discharge (DOD) of a battery with size B is not 100%, the installed battery size is B/DOD.

[3]To incorporate the battery charge-discharge energy efficiency factor $0 \leq \eta \leq 1$ to our formulation, we can simply replace C in our formulation with $C\eta$.

Name	*Description*
Power outage	Electricity from the grid is unavailable
Loss of power	An outage period with battery being empty
B	Storage battery capacity
C	Battery charging rate
ϵ^*	Target loss of power probability
$x(t)$	Grid availability at time t
$x^c(t)$	Grid unavailability at time t
$d(t)$	Power load at time t
$d^e(t)$	Effective power load to battery at time t
$b(t)$	Battery charge level at time t
$b^d(t)$	Battery deficit charge at time t
$l(t)$	Amount of loss of power at time t
$\mathcal{G}$	Statistical sample path envelope
ε_g	Bounding function for sample path envelope

TABLE I: Notation



(a) Grid available

(b) Power Outage

Fig. 3: Storage battery model before and after transformation

A. Loss of power formulation

As discussed in Section II, due to major differences in system models we cannot apply the loss of power formulation in [7] to our problem. However, the following modeling approach converts our system model to theirs, allowing us to use their loss of power formulation (see Figure 3).

Consider the input and output processes to the battery separately for the cases where the grid is available ($x(t) = 1$) and unavailable ($x(t) = 0$). If $x(t) = 1$, the arrival energy process to the battery at any time instant t is a constant, C, and the departure energy process is zero. If $x(t) = 0$, the arrival energy process is zero and the departure energy process is $d(t)$. The battery state-of-charge does not change if we assume that the real arrivals and departures to the battery are both shifted by the same constant at any time. Therefore, we can assume C and $d(t) + C$, respectively, as the arrival and departure processes when $x(t) = 0$ (instead of 0 and $d(t)$) and have the same battery state of charge. Combining the two cases, $x(t) = 0$ and $x(t) = 1$, with the above substitution, we can assume that the battery is always charged with the rate C and discharged by the *effective demand* d^e defined as:

$$d^e(t) = [d(t) + C](1 - x(t)) = [d(t) + C]x^c(t) \quad (10)$$

which is the portion of the demand that the battery must serve. Using the above transformation and Eq. (6), we have:

$$b^d(t) = \min(B, [b^d(t-1) + d^e(t) - C]_+) \ . \quad (11)$$

The power loss at time t is given by $l(t) = [b^d(t-1)+d^e(t)-C-B]_+$, which can be mapped to traffic loss in a buffer with size B, service rate C, and input traffic d^e. Hence, we can use Eq. (4) to describe the loss of power process as follows:

$$l(t) = \min_{0\leq u\leq t-1}\Bigg(\max_{u\leq s\leq t-1}([D^e(s,t) - C.(t-s) - k(t)]_+, \\ [D^e(u,t) - C.(t-u) + k(u) - k(t))]_+\Bigg) \ , \quad (12)$$

where D^e is the cumulative version of the effective demand (Eq. (10)) and k is as expressed in Eq. (5).

B. Battery sizing to eliminate the use of a genset

The exact loss description from Eq. (12) is difficult to use in practice. Instead, we use the following upper bound (from [26]) to derive an upper bound on the loss probability:

$$l(t) \leq \min\big([d^e(t) - C]_+, \\ \max_{0\leq s\leq t-1}([D^e(s,t) - C(t-s) - B]_+)\Big) \quad (13)$$

$$= \min\big(d(t)x^c(t), (\ . \\ \max_{0\leq s\leq t-1}([D^e(s,t) - C(t-s) - B]_+)\Big) , \quad (14)$$

where in Eq. (13) two specific values for u in the minimization of Eq. (12) are chosen: $u = t-1$ and $u = 0$. Eq. (14) uses the definition of $d^e(t)$ and that $d(t) > 0$. This inequality can be used to compute a probabilistic upper bound on the loss of power for our problem. We now state the following Lemma.

Lemma 1 (Amendment to Theorem 1 [7])**.** *Suppose that an unreliable grid uses a battery of size B with charging rate C to serve a demand. Suppose also that $x^c(t)$ represents the grid unavailability at time slot t ($x^c(t) = 1$ if the grid is unavailable and $x^c(t) = 0$, otherwise) and $\mathcal{G}$ is a statistical sample envelope for process D^e with bounding functions ε_g in the sense of Eq. (7). Then, the loss of power probability satisfies the following*

$$\Pr\{l(t) > 0\} \leq \min\Bigg(\Pr\{x^c(t) > 0\}, \\ \varepsilon_g\left(B - \max_{\tau\geq 0}(\mathcal{G}(\tau) - C\tau)\right)\Bigg) \ . \quad (15)$$

The proof can be found in the accompanying technical report [27].

We can use Lemma 1 to compute the minimum battery size satisfying a target loss of power probability ϵ^* by bounding $\Pr\{l(t) > 0\}$ with ϵ^*. We observe that the first term, $\Pr\{x^c(t) > 0\}$, in the minimum expression of Eq. (15) is independent of the battery size. We can therefore set battery size to be zero whenever the first term forms the minima. Intuitively this means that there is no need of a battery if the probability of power outage is less than ϵ^*.

From Lemma 1, If $\mathcal{G}$ is a statistical sample path envelope on the effective demand in the sense of Eq. (7) with bounding function ε_g, then using Eq. (15), we get

$$\min\Bigg(Pr\{x^c(t) > 0\}, \varepsilon_g\left(B - \max_{\tau\geq 0}(\mathcal{G}(\tau) - C\tau)\right)\Bigg) \leq \epsilon^* \\ \implies B \geq \left(\max_{\tau\geq 0}(\mathcal{G}(\tau) - C\tau) + {\varepsilon_g}^{-1}(\epsilon^*)\right) I_{(\Pr\{x^c(t)=1\}>\epsilon^*)} \quad (16)$$

where I_{expr} is the indicator function, which is 1 if $expr$ is true and is 0, otherwise.

C. Battery sizing to limit the carbon footprint of a genset

For the battery-genset hybrid system, at any instant that the grid is unavailable, the demand can be either served by the battery or by the genset. It is desirable to reduce carbon emission from the genset by using the battery to store 'greener' energy produced by the grid and consuming it when an outage occurs. The optimum scheduling algorithm between the battery and the genset must minimize the carbon emission from the genset while keeping the loss of power below an acceptable threshold. Scheduling is trivial if the genset size is larger than the maximum (worst-case) demand load (i.e., $\max_t d(t)$). This is because the demand can be always met by using the genset and carbon emission is minimized by always scheduling energy from the battery whenever it is not empty. The scheduling, however, is non-trivial if the genset size is smaller than the maximum demand load (See [27] for a more detailed discussion).

For simplicity, in this work we assume that the genset capacity is large enough to meet the maximum aggregate load (Assumption 3 in Section IV). This is reasonable because the marginal cost of increasing genset capacity is small compared to the storage battery.

The objective in battery sizing in the presence of genset is to keep the carbon emission below a certain threshold. The carbon emission is proportional to the total demand that cannot be served by the battery when the grid is unavailable, i.e.,

$$\text{carbon emission} \sim \sum_t l(t) \ . \quad (17)$$

Using the analogy between a queueing system and the distribution power system, this quantity corresponds to the total loss in a finite-buffer queue. In spite of extensive efforts, the problem is still open for non-Poisson arrivals. The complexity of the problem arises from the fact that the total loss is a function of the number and length of the busy periods that occurs in a time interval. Liu and Cruz [28] show that a probabilistic upper bound on the total loss must account for the numbers and lengths of the busy periods. This leads to cumbersome formulations that cannot be used in practice. Here we compute an upper bound on the expected value of the total loss (carbon emission) in a time interval of size T using Eq. (14) as follows:

$$E\left[\sum_{t=1}^{T} l(t)\right] = \sum_{t=1}^{T} E[l(t)]$$

$$\leq \sum_{t=1}^{T} E\Big[\min\Big(d(t)x^c(t), \max_{0\leq s<t}\left([D^e(s,t) - C(t-s) - B]_+\right)\Big)\Big] \quad (18)$$

$$\leq \sum_{t=1}^{T} \min\Big(E\left[d(t)x^c(t)\right], E\left[d(t)x^c(t) I_{\max([D^e(s,t)-C(t-s)-B]_+)>0}\right]\Big) \quad (19)$$

$$\leq \sum_{t=1}^{T} \min\left(E\left[d(t)x^c(t)\right], E\left[d(t) I_{\max([D^e(s,t)-C(t-s)-B]_+)>0}\right]\right) \quad (20)$$

$$\approx \min\Big(\sum_{t=1}^{T} E\left[d(t)x^c(t)\right], \Pr\{\max\left([D^e(s,t) - C(t-s) - B]_+\right) > 0\}. \sum_{t=1}^{T} E[d(t)]\Big), \quad (21)$$

where we use Eq. (14) to obtain Eq. (18). The first term in Eq. (19) is trivial as it is the first term in the minima of the previous line. The second term in Eq. (19) only accounts for the sign of the second term in the minima of Eq. (18) and sets the whole expression to zero if that term is zero, otherwise it returns the first term in Eq. (18). The next line uses the fact that $0 \leq x^c(t) \leq 1$. We assume that the two processes in the second term in Eq. (20) are independent to derive Eq. (21). This assumption holds for statistically independent increments processes, which is widely assumed in the literature (e.g., Kelly [29]). In addition, we numerically find in Section VI-B that the inaccuracy due to this approximation step is small. Numerically, we also find that the upper bound used to derive Eq. (19) from Eq. (18) is quite tight. More precisely, we observe that Eq. (14) evaluates to its first term if the second term is positive.

Suppose $\mathcal{G}$ is a statistical sample path envelope on D^e with bounding function ε_g. Then, from Eq. (21), the following is the desired upper bound on the average carbon emission:

$$\text{average carbon emission} \leq \min\Big(\sum_{t=1}^{T} E\left[d(t)x^c(t)\right], \varepsilon_g\left(B - \max_{\tau\geq 0}(\mathcal{G}(\tau) - C\tau)\right). \sum_{t=1}^{T} E[d(t)]\Big) \quad (22)$$

VI. Numerical Study

In this section, we compare our battery sizing approach with the optimal for traces of electrical loads collected over 12 months from 4500 homes in Ireland [8]. The effective demand (Eq. 10) is derived from randomly selected sets of 100 homes in this dataset. Since we could not find real datasets for grid availability ($x(t)$), we use a two state Markov-modulated ON-OFF process to simulate it. The transitions between ON→OFF and OFF→ON are exponentially distributed with respective rates $\mu = 1\ hr^{-1}$ and $\lambda = \frac{1}{11}\ hr^{-1}$. This corresponds to an average of two power outages in a day with the average duration of a power outage being one hour, which is common for developing countries such as India. Moreover, unless otherwise stated, the target violation probability in examples is assumed to be one day loss of power in ten years, i.e. $\epsilon^* = 2.7 * 10^{-4}$, and battery charging rate $C = 100$kW.

We use leaky-bucket as the statistical sample path envelope (defined in Eq. (7)) on D^e, i.e. $\mathcal{G}(t) = a + bt$ for some a and b, and fit either a hyper-exponential or a Weibull distribution on ε_g.

A. Battery sizing for eliminating genset

In this section we evaluate our analytical results on the required battery size to eliminate the use of genset. The accuracy of our analysis depends on two major issues: (1) how accurately our formulation can compute the loss of power or carbon emission values assuming that there is no inaccuracy in modeling the effective demand, i.e. inaccuracy till Eq. (14), and (2) how accurately we model the effective demand using envelopes.

We study the following cases (more details in [27]):

- *Empirical optimal*: This is the battery sizing obtained by using the trace of effective demand as the input to the exact recursive loss of power description in Eq. (3). The battery size obtained by this method is the smallest value that satisfies the target loss of power probability for exactly known future demand.
- *Our bound*: Using Lemma 1 for loss of power formulation, we find the least battery size that satisfies the target loss of power probability (as given in Eq. (16)).
- *Ideal D^e model*: Here, we want to remove the inaccuracy induced by envelope fitting on D^e from the inaccuracy caused by the loss of power formulation. For this reason, we use traces of D^e in Eq. (14) and compute the numerical optimal for a target loss of power probability.

Figure 4 compares the storage size as a function of the target loss of power probability ϵ^*. To accurately model the fluctuations of the demand we consider battery sizing for different weather seasons by classifying the demand dataset into three seasons: Winter (December-March), Summer (April-July), and Autumn (August-November). We compute the battery size for each season separately. Note that a battery size that satisfies the loss of power requirements throughout the year would be the maximum of the battery sizes computed for each season. Hence, we use the winter season for all other figures.

We first observe from Figure 4 that for one day in ten years target loss of power probability (i.e., $2.7 * 10^{-4}$), 'Ideal D^e model' is within 10% of the 'empirical optimal' implying that our loss of power formulation is reasonably tight. We also notice that different seasons can have significantly different battery size requirements (up to 35%) probably due to heating appliances used in cold weather.

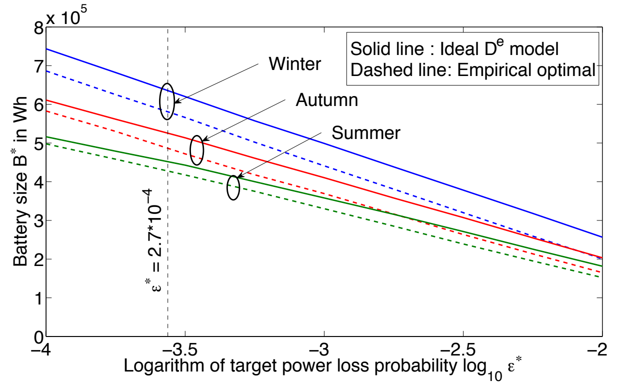


Fig. 4: **Accuracy of loss of power formulation:** *Empirical optimal* and *Ideal D^e model* battery sizes as a function of target violation probability in three seasons

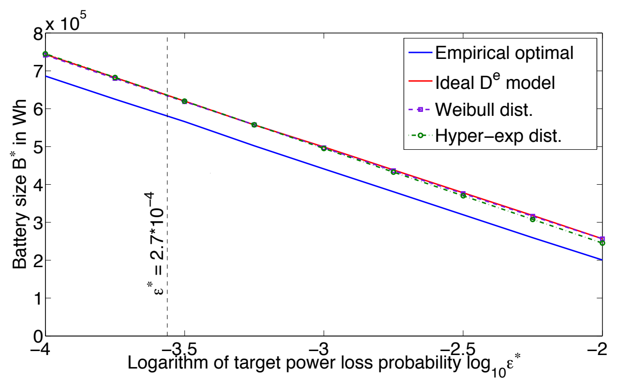


Fig. 5: **Accuracy of D^e modeling:** Minimum battery size B^* as a function of violation probability ϵ^*.

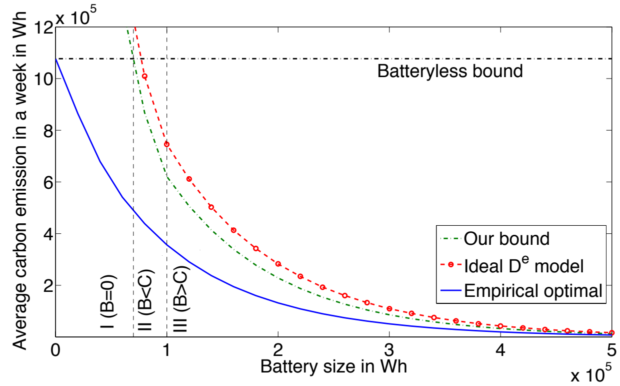


Fig. 6: **The impact of battery size on carbon emission**

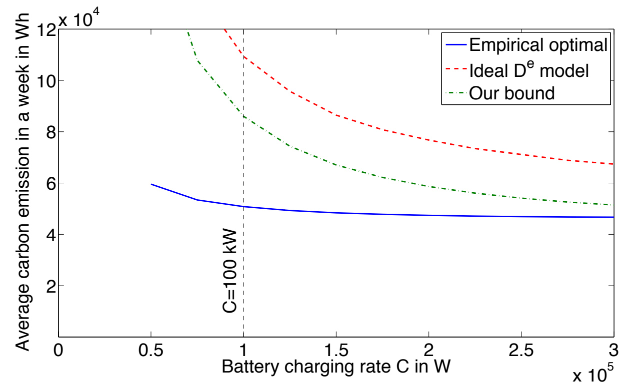


Fig. 7: **The impact of charging rate on carbon emission:** Carbon emission from genset as a function of battery charging rate C, and constant battery size $B = 3 * 10^5 Wh$

We evaluate the performance of the fitting technique in modeling D^e by comparing the battery sizes obtained by this model and those from 'empirical optimal' and 'Ideal D^e model'. In fact, the difference between the battery sizing by envelope fitting with that of 'Ideal D^e model' indicates the accuracy of fitting; recap that the 'Ideal D^e model' shows the battery sizing scheme when the D^e modeling inaccuracy is eliminated.

Figure 5 compares the battery sizes computed using different methods as a function of the violation probability. This example indicates that our D^e modeling and loss of power formulations are quite tight.

We also compare our loss of power formulation to Kesidis [9] and MSM [30] and find that our approach outperforms them. Due to space constraints we refer the reader to [27] for details.

B. Battery sizing for limiting genset carbon footprint

In this section, we compare three approaches to find the minimum battery size that keeps the carbon footprint of a given genset below a certain threshold.

The plots of this section show the following curves:

- *Empirical optimal*: Similar to the 'Empirical optimal' earlier, we use the trace in Eq. (3) and compute the exact carbon emission for our dataset.
- *Our bound*: This is the upper bound on carbon emission from Eq. (22) using envelope fitting an envelope $\mathcal{G}(t) = at + b$ for some a and b on D^e with hyper-exponential distribution on ε_δ.
- *Ideal D^e model*: We remove the inaccuracy induced by independence assumption in Eq. (21) by using the dataset trace in Eq. (20) to compute upper bounds on carbon emission.

We compute the carbon emission as a function of the battery size for charging rate of $C = 100$kW in Figure 6. For any given carbon emission threshold, one can use this plot to compute the required battery size. The plot is divided into three regions I, II, and III. Region I corresponds to the case where Eq. (21) is evaluated to $\sum_{t=1}^{T} E[d(t)x^c(t)]$, which is the battery-less scenario. If the carbon emission target threshold is as large as any value in this region we do not need a battery. Region II corresponds to the case when $C > B$. If the target carbon emission threshold falls into this region, one should choose the battery size to be equal to the energy generated in one time slot with the charging rate, i.e., $B = C$. Finally, Region III corresponds to the case of $C < B$ for which the curves in that region must be used to size the battery.

From Figure 7 we can see that our bound is loose for smaller charging rates and becomes more accurate for larger charging rates. Even for a relatively small charging rate like 100kW, we find that our bound is within a factor of 1.7 from the 'empirical optimal'. We can observe from Figures 6-7 that the carbon emission using Eq. (22) are slightly below the 'Ideal D^e model'. This is because of the independence assumption in Eq. (21).

VII. CONCLUSIONS

Motivated by the need of reducing carbon footprint of diesel generator operation to mitigate power outages, we present an analytical technique based on the stochastic network calculus for a representative set of homes to choose a battery size to reduce carbon emission from a genset. We solve the problem in two parts. First, we study the problem of eliminating the use of genset and find the smallest battery size needed to ensure a given target loss of power probability. Numerical evaluations show that the sizing using our methodology is within 10% of the minimum battery size required had the future load been exactly known. In contrast, the battery size computed using classical methods is far larger than necessary.

Secondly, we study the trade-off between the size of battery and genset carbon emission. For a given battery size, our computation of the carbon emission is within a small factor (1.7) of the value obtained through numerical evaluation. This allows us to find the battery size needed to limit the carbon footprint of a diesel generator. Given that prior work in the area of teletraffic analysis has had limited success in computing upper bounds on the total loss of buffer for non-Poisson arrivals, we believe that our work is of general interest, even in the area of teletraffic analysis.

Our results are necessarily limited by the lack of demand and outage data from developing countries. We model the demand from a neighbourhood of homes in a developed country by the demand of 100 randomly selected Irish homes and we assume that outages are modeled by a two-state Markov model. These limit the strength of our numerical results. Nevertheless, our general approach can be used to study real datasets when they are available.

We are also interested in eliminating the need to assume statistically independent increments when studying the total loss of power. Finally, an interesting related problem is to work with more complex models of backup generation to analytically study how a battery can improve its efficiency.

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