Firming Solar Power

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ABSTRACT

The high variability of solar power due to intrinsic diurnal variability, as well as additional stochastic variations due to cloud cover, have made it difficult for solar farms to participate in electricity markets that require pre-committed constant power generation. We study the use of battery storage to 'firm' solar power, that is, to remove variability so that such a pre-commitment can be made. Due to the high cost of storage, it is necessary to size the battery parsimoniously, choosing the minimum size to meet a certain reliability guarantee. Inspired by recent work that identifies an isomorphism between batteries and network buffers, we introduce a new model for solar power generation that models it as a stochastic traffic source. This permits us to use techniques from the stochastic network calculus to both size storage and to maximize the revenue that a solar farm owner can make from the day-ahead power market. Using a 10-year of recorded solar irradiance, we show that our approach attains 93% of the maximum revenue in a summer day that would have been achieved in daily market had the entire solar irradiance trace been known ahead of time.

Categories and Subject Descriptors

C.4 [**Performance of Systems**]: Modeling techniques

Keywords

solar power modelling, electricity market

1. INTRODUCTION

The intermittent nature of solar energy hinders its fast penetration in the day-ahead electricity market. Using storage to firm solar power is one of the most promising solutions. An important and complex problem in this area is maximizing revenue in the day-ahead market for a battery-equipped solar farm given the weather forecast and the battery size. This problem has been explored in the literature for wind power, but not solar power, which is different in that it exhibits stochastic fluctuations modulating a deterministic diurnal variation. Our work proposes a highly accurate technique for solar firming as illustrated by numerical examples.

Our key contributions are:

1. We provide a new stochastic model for solar output power which obviates the shortcomings of existing models. It lays the foundation of many analyses that require the stochastic characterization of the solar energy including storage sizing, and revenue maximization in the day-ahead market.

2. We use this model along with existing power loss formulations from the stochastic network calculus to provide solutions to the revenue maximization in the day-ahead market.

3. Using a large real data set consisting of the solar irradiance measurements gathered over 10 years, we numerically show that our analyses are quite accurate.

2. SOLAR POWER MODELLING

Solar power is measured by irradiance, which is the total power incident on a unit area, and is referred to as the global irradiance i_g. The output electricity power from a photovoltaic (PV) panel is $\alpha_{pv}I_g$[1], where α_{pv} is the product of PV area and PV panel efficiency in converting solar energy into electricity. Our aim is to characterize $\alpha_{pv}I_g$.

Fluctuations in i_g consist of two elements: (1) i_{det}: the deterministic variation in diurnal irradiance which mainly occurs due to the position of the sun in the sky (also called clear-sky irradiance) and (2) $i_{det}(t) - i_g(t)$: the high frequency fluctuations due to the effect of clouds which modulate the clear-sky irradiance.

Existing clear-sky irradiance models accurately capture variations in i_{det} as a function of location, date, and time. Hence, the fluctuation of $i_g(t)$ at a given time instant t is limited to $i_{det}(t) - i_g(t)$. Existing models, however, do not accurately capture short term fluctuations. To be more precise, the best known models (e.g., step changes [3, 4] and wavelet-based analysis [2]) require stationarity and for this reason they choose to model the clear-sky index ($\frac{i_g}{i_{det}}$) instead of $i_g(t) - i_{det}(t)$ assuming that clear sky index (CSI) is stationary which is not true in practice.

To characterize $\alpha_{pv}I_g$, we seek to find a statistical sample path lower envelope β^l with bounding function ε^l which satisfies the following for any time $t \geq 0$ and any $x \geq 0$

$$\mathbb{P}\left\{\sup_{s \leq t}\{\beta^l(t-s) - \alpha_{pv}I_g(s,t)\} > x\right\} \leq \varepsilon^l(x). \qquad (1)$$

Note that we do not require stationarity. We model the following stochastic process instead of CSI:

$$i_{aux}(t) = i_{det}(t) - i_g(t) + \text{offset}, \qquad (2)$$

where offset is a constant chosen such that $i_{aux}(t)$ is always positive. One can simply set it to $\max_t(i_g(t) - i_{det}(t))$.

We model fluctuations in I_{aux} using an approach originally developed for teletraffic modelling by determining two functions $\mathcal{G}_{aux}^u$

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[1]We use capital letters to denote cumulative processes, i.e., $I_g(t) = \sum_{\tau=1}^{t} i_g(\tau)$.
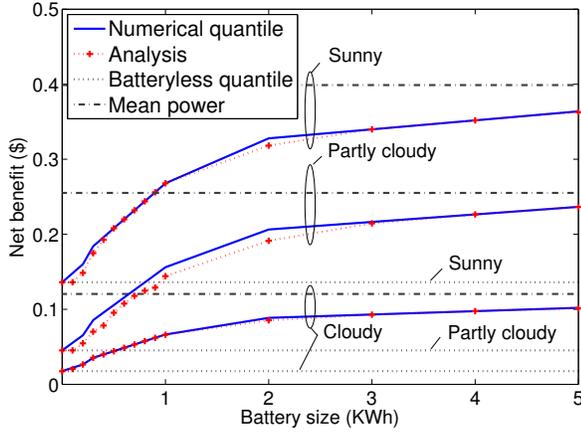
Figure 1: Day-ahead dispatch problem: net benefit as a function of the battery size in summer days in site C1 with $p = 58.51\$/MWh$, $c = 55.51\$/MWh$, $\alpha_{pv} = 1$, $T = 8$ hr, and $T_1 = 8$ am.

and ε^u such that

$$\mathbb{P}\left\{\underbrace{\sup_{s \leq t}\{I_{aux}(s,t) - \mathcal{G}^u_{aux}(t-s)\}}_{:=Y(t)} > x\right\} \leq \varepsilon^u(x) \qquad (3)$$

for any $x \geq 0$ and $t \geq 0$. Note that ε^u can be chosen to be the complementary cumulative distribution function (CCDF) of Y. That is, $\varepsilon^u(x) = \int_x^\infty f_Y(y)dy$, where f_Y is the probability density function of Y.

Given $\mathcal{G}^u_{aux}$ and ε^u from Eq. (3) and combining it with Eq. (2), we are able to provide a sample path lower envelope β^l on $\alpha_{pv}I_g$ with bounding function ε^l in the sense of Eq. (1).

3. POWER LOSS CALCULATIONS

Using the above definitions and given a lower sample path envelope on the solar irradiance, we improve the power loss formulation from [5] using the following lemma.

LEMMA 1. *Suppose that an intermittent power source with process $\mathcal{S}$ is fed to a battery with size C and the battery is used to provide a constant output power $\overline{P}$. There is a statistical sample path lower envelope β^l on the intermittent energy source with bounding function ε_l in the sense of Eq. (1). Suppose that ε^{bl} is a constant which satisfies*

$$\forall \tau \geq 1: \qquad \mathbb{P}\{\overline{P} > \mathcal{S}(\tau-1,\tau)\} \leq \varepsilon^{bl} . \qquad (4)$$

If the initial state of deficit battery charge[2] is B_0^d, then the power loss at time t satisfies the following

$$\mathbb{P}\{l(t) > 0\} \leq \min\left(\varepsilon^{bl}, \varepsilon_l\left(C - B_0^d - \sup_{0 \leq \tau \leq t}(\overline{P}\tau - \beta^l(\tau))\right)\right) . \qquad (5)$$

4. ELECTRICITY MARKET

Suppose that a power supplier wants to trade the next day's available power in the electricity market for a time interval of size T, where the start time of the contract is T_1. Energy can be traded as

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[2]defined as the amount of energy needed to fully charge the battery at a given time instant.

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constant-power bids for time slots of size T_s (totalling $\frac{T}{T_s}$ contracts per day) and the supplier can propose a different constant power at each time slot. The supplier earns $\$c$ for each watt-hour it is scheduled for. During the day of operation, if the supplier cannot provide its scheduled power for a time interval of size larger than T_u, he is penalized $\$p$ for each watt-hour under-power. Thus, T_u is the time resolution (time unit) needed for the revenue maximization; note that there are $\frac{T_s}{T_u}$ time units in a time slot.

For a given battery size and lower bound sample path envelope on solar irradiance, we can compute the guaranteed output power $\overline{P}_{\varepsilon^*}(\tau)$ watt in τ'th time slot of the next day with loss of power less than $\varepsilon^*_{\overline{P}}(\tau)$ using Lemma 1. Denoting by P the actual available power, the revenue maximization is given by

$$\text{DA max revenue} \geq \max_{0 \leq \varepsilon^* \leq 1} \sum_{\tau=T_1}^{\frac{T+T_1}{T_s}} \Big(c\overline{P}_{\varepsilon^*}(\tau) $$
$$- p\frac{T_s}{T_u}\varepsilon^*_{\overline{P}}(\tau)[\overline{P}_{\varepsilon^*}(\tau) - P(\tau)]_+ \Big)T_s . \qquad (6)$$

5. EVALUATION

We use the the dataset from SGP-$C1$ permanent site of Atmospheric Radiation Measurement (ARM) program [1] for a large time interval of 10 years (from 2002 to 2011) to numerically evaluate the accuracy of our analysis; the results are shown in Fig. 1, illustrating that our analysis closely matches numerical simulations across a variety of weather conditions.

6. CONCLUSION

Our work provides an accurate approach, based on the stochastic network calculus, to compute probabilistic bounds on solar production. We believe that our solar power model captures solar generation variability over multiple time scales more accurately than the best-existing models. Using this approach, we pose and solve an optimization problem where we seek to maximize the expected revenue of a solar farm operator participating in a day-ahead market. Our framework can also be used to size a battery such that the overall revenue during battery life time is maximized. We have evaluated our analysis on ten year's traces of solar output and find that our analysis closely approximates the results found from an exact numerical evaluation.

This work assumes perfect batteries. In ongoing work, we are using teletraffic approaches to model battery imperfections.

7. REFERENCES

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