Battery Provisioning and Scheduling for a Hybrid Battery-Diesel Generator System

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ABSTRACT

Diesel generators (gensets) are commonly used to provide a reliable source of electricity in off-grid locations. Operating a genset is expensive both in terms of fuel and carbon footprint. Because genset efficiency increases with offered load, this expense can be reduced by using a storage battery to ensure that a genset always runs at full load, charging and discharging the battery as necessary. However, the cost of batteries requires us to size them parsimoniously and operate them efficiently. We, therefore, study the problem of provisioning and optimally managing a battery in a hybrid battery-genset system. To aid in sizing a battery, we analytically study the trade-off between battery size and carbon footprint. We also formulate the optimal scheduling of battery charging and discharging as a mixed-integer program, proving that it is NP-hard. We then propose a heuristic online battery scheduling scheme that we call *alternate* scheduling and prove that it has a competitive ratio of $\frac{k_1 G/C + k_2 T_u}{k_1 + k_2 T_u}$ with respect to the offline optimal scheduling, where G is the genset capacity, C is the battery charging rate, k_1, k_2 are genset-specific constants, and T_u is the duration of a time step. We numerically demonstrate that alternate scheduling is near-optimal for four selected industrial loads.

1. INTRODUCTION

Although modern society is critically reliant on electric power, there are still many locations where there is no electrical grid access. Extending the grid to such locations (such as remote villages) is sometimes difficult (as in rural India) or infeasible (as in Northern Ontario) [4]. Grid unavailability is also a problem for mobile base stations and mining industries located in remote areas [5]. To compensate for grid unavailability, off-grid communities install a diesel generator (genset) to meet their demands. These gensets use expensive diesel fuel and have a high carbon footprint. Adding a battery to the genset to create a hybrid battery-genset system can reduce both fuel costs as well as the footprint. We study how to size and operate a hybrid battery-genset system to reduce the carbon footprint of the genset in the context of off-grid locations, where all demands must ultimately be met by the diesel generator. Specifically, we focus on how an off-grid community or small-scale industry that already uses a genset to meet its entire demand can simultaneously reduce its carbon footprint and fuel costs by converting to a battery-genset hybrid system.

Our work is based on the key insight that a battery can increase genset efficiency. Genset efficiency–the ratio of energy production to fuel consumption–is known to be the largest when the genset operates close to its capacity (also called rated or nominal power) G [6]. For off-grid deployments, a genset is usually sized to meet the occasional demand peaks, which causes it to typically operate at low efficiencies. Moreover, continued operation of genset to meet small demands can lead to engine damage [3]. A battery improves genset efficiency in two ways: (a) a battery can entirely meet *small* loads so that the genset can be turned off, (b) for *large* loads, the genset can be run to simultaneously meet the load and recharge the battery, thereby, running closer to its full capacity.

Given this context, we focus on the following two problems. First, we study the tradeoff between the expected carbon emission from a hybrid battery-genset system and the battery size. Second, given the battery characteristics, we study how to meet the demand from the battery, genset, or both (i.e., the *scheduling* problem) such that the carbon emission is minimized. Our approach is analytical, with additional insight gained from numerical examples.

The key contributions of our work are:

1. We study the tradeoff between the size of a battery and the expected carbon emission from a hybrid battery-genset system in the context of an off-grid deployment. We show that carbon emission depends greatly on the battery charging rate and is almost insensitive to the battery size once the battery size exceeds a small threshold.

2. We analytically study the genset-battery power scheduling problem to minimize carbon emission. We show that although the general problem is NP hard, there exists an online scheduling algorithm, *alternate scheduling*, that performs close to the offline optimal scheduling.

3. We use case studies of measured electricity consumption data from four commercial loads in Ontario, Canada, to numerically verify our analytical results.

The remainder of the paper is organized as follows. We will briefly discuss the literature and motivate the problem in Section 2. Our system model is explained in Section 3. We then study the battery scheduling and provisioning in Section 4. We evaluate our analysis numerically in Section 5 and conclude our paper with some discussions in Section 6.

2. MOTIVATION AND RELATED WORK

It is well-known that gensets have higher efficiencies when operated close to their rated capacity [6]. Nonetheless, efficiency-improving algorithms to date rely primarily on numerical analysis rather than analytical modeling [1, 2]. They use the typical daily load profile to define a non-linear optimization problem and solve it using numerical techniques, such as simulated-annealing, to schedule power from the battery and the genset. As one of the most promising methods to achieve high efficiency for genets, the benefits of using the hybrid battery-genset system have also been discussed in [5].

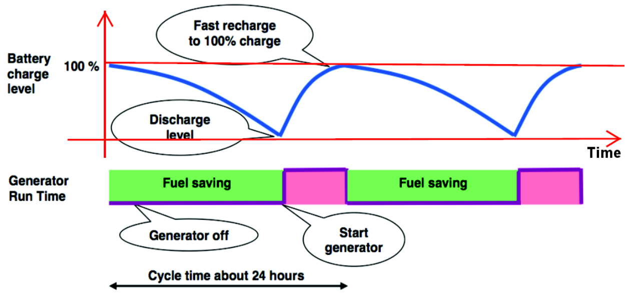

Figure 1: Using a battery to decrease genset fuel consumption [5]

Name	*Description*
G	Maximum genset power production rate
B	Battery storage capacity
C	Maximum battery charging rate
$d(t)$	Energy demand in time slot t
$b(t)$	Battery level of charge at time t
$I_g(t)$	Indicator variable for genset at time t
T	Total time interval of interest
T_u	Time unit
τ	Expected time needed to deplete a fully-charged battery
ρ_d	Long-term average rate of the demand process

Table 1: Notation

Consider a discrete-time system where time is divided into time slots of equal duration T_u. The carbon emission rate from a genset is proportional to its fuel consumption rate, which, in turn, can be expressed as an affine function of the per time-slot demand $d(t)$ as follows: [1, 6]:

$$\text{Genset fuel consumption rate (t)} = (k_1 G + k_2 d(t)) I_g(t) \quad (1)$$

where k_1 and k_2 are genset-specific constants and $I_g(t)$ is an indicator function which is one if genset is on at time slot t and zero, otherwise. The total fuel consumption by a genset-only system (i.e., with no battery) during time slots $[1, T]$ is, therefore,

$$\text{Genset-only fuel consumption} = k_1 G \sum_{t=1}^{T} I_{d(t)>0} + k_2 \sum_{t=1}^{T} d(t) \ , \quad (2)$$

where I_{expr} is an indicator function which is one at time instant t if $expr$ is true and is zero, otherwise.

Eq. (2) suggests that if a genset is the only source of energy, then we cannot have any improvement in the second term, even with the addition of a battery, because all the demand must eventually be served by the genset. However, the first term can be reduced by using a battery to shape the load from $d(t)$ to $d'(t)$ to minimize the total carbon footprint of a hybrid battery-genset system, given by

$$\text{Hybrid-system fuel consumption} = k_1 G \sum_{t=1}^{T} I_{d'(t)>0} + k_2 \sum_{t=1}^{T} d(t) \ . \quad (3)$$

To repeat, the reduction in fuel consumption in the hybrid system comes from reducing the first term, which is the total number of time slots where $d'(t) > 0$ that is, the genset is on. Thus, minimizing Eq. (3) is equivalent to minimizing the total time slots that the genset is on. Figure 1 illustrates this idea and formally it can be stated as follows:

OBSERVATION 1. *Consider a hybrid battery-genset system where the battery is initially empty (this ensures that we do not use external energy sources other than the genset). For a given genset and battery size, the problem of minimizing genset fuel consumption is equivalent to the problem of minimizing the number of time slots for which the genset is turned on.*

The maximum achievable gain from adding a battery to a genset is given by the following example. Suppose the demand is slightly greater than zero for the entire T time slots. In absence of a battery, the genset must be on for all the T slots and hence the first term will be $k_1 GT$. However, in presence of a battery, the genset will be on, only for the first time slot to meet the demand and charge the battery. For the remaining time slots, entire demand can be met by discharging the battery. Hence, the first term is only $k_1 G$, which is T times smaller.

Note that a hybrid battery-genset system operates in one of the following three modes to serve the demands [1, 2]:

(i) **Demand met by battery:** This mode has zero carbon footprint.

(ii) **Demand met by genset, with excess capacity used to charge the battery:** This mode causes carbon emission.

(iii) **Demand met by both battery and genset in parallel:** This mode allows us to size the genset capacity smaller than the peak load.

The optimal schedule determines the mode of operation at each time step to minimize total carbon emission. Specifically, it schedules the battery to meet small demands using mode (i). Moreover, it schedules operations in mode (iii) to reduce the genset size by using the battery charge to meet rare huge bursts of demands. This increases efficiency because the genset can now operate closer to its capacity. If the demand is high (but less than the genset capacity G) it is most efficient to use genset to meet both the demand and recharge the battery, using mode (ii).

We now study how to size the battery and schedule operating modes to minimize total fuel consumption. We begin by formalizing the system model.

3. SYSTEM MODEL

We assume a discrete time model with time unit T_u, where time-slot t represents the time interval $[(t-1)T_u, tT_u)$. The length of the time slot should neither be very small, because this increases the size of the numerical computation, nor very large such that we lose accuracy of approximating a continuous system. Time units cannot be smaller than the on/off switching speed of gensets, which is typically around a minute. Therefore, we suggest a time unit T_u of about $5-15$ mins.

We assume a perfectly efficient battery, where the charging rate is upper bounded by a constant C ($\leq G$) and is independent of the battery size B and also of its state of charge [1]. Denote by $d(t)$ the energy demand in time slot t and by $b(t)$ the battery state of charge at the end of time slot t. We define demand profile $\mathcal{D} = \{d(1), d(2), \ldots, d(T)\}$ to be the set of demands for T consecutive time slots. To ensure that there is no external source of energy, we

[1] We are aware that this is a very simple battery model: real storage technologies have many other constraints that we have elided for the sake of simplicity.

assume that the battery state of charge at the beginning of slot 1 and at the end of slot T is zero, i.e. $b(0) = b(T) = 0$.

Using the above notation, the battery state of charge at the end of time slot t is given by the following.

(i) **Demand met by battery:** The battery is discharged to meet the demand while the genset is turned off

$$b(t) = [b(t-1) - d(t)]_+ \quad (4)$$

(ii) **Demand met by genset:** The genset is turned on and assuming genset is large enough to meet the demand ($GT_u > d(t)$), it simultaneously meets the demand and charges the battery. Any demand greater than the capacity of genset cannot be met in this mode.

$$b(t) = \min\{B, [b(t-1) + \min\{GT_u - d(t), CT_u\}]_+\} \quad (5)$$

(iii) **Demand met by both battery and genset in parallel:** The genset is turned on and the battery is discharged simultaneously to meet the demand. Genset is not enough to meet the demand in this mode, i.e., $d(t) > GT_u$.

$$b(t) = [b(t-1) - (d(t) - GT_u)]_+ \quad (6)$$

Given a demand profile $\mathcal{D}$, a battery size B, and a charging rate C, a genset scheduling strategy is an algorithm that computes a unique *schedule* $\mathcal{S} \subseteq \{1, 2 \ldots, T\}$ that denotes the set of time slots when the genset is turned on to meet the demand and charge the battery. We sometimes also refer to the algorithm by set $\mathcal{S}$ to simplify notation. The strategy is called online (offline) if the decision about time t is independent (dependent) of demands after time t.

We define the following scheduling algorithms:

1. **Feasible scheduling strategy**:
For any demand profile $\mathcal{D}$, battery size B, and charging rate C, we define a genset scheduling strategy $\mathcal{S}$ to be feasible if the following holds:
   (a) The genset-hybrid system is always in one of the three modes of operation and follows the battery state of charge equations.
   (b) There is no loss of load and the battery state of charge at the beginning of time slot 1 and at the end of time slot T is zero.
2. ***Genset-only* scheduling strategy**:
The genset is always on and the battery does not operate at all (or does not exist), i.e. $\mathcal{S} = \{1, 2, \ldots, T\}$.
3. ***Alternate* scheduling strategy**:
A scheduling where the system alternates between running the genset until the battery is completely charged to its capacity B and using only the battery to meet the demand until it is fully depleted.

Given a battery size B and a charging rate C, for any genset scheduling strategy $\mathcal{S}$ we define the competitive ratio α (≥ 1) as

$$\sup_{\mathcal{D}:\mathcal{S} \text{ is feasible}} \frac{\text{Fuel consumption by } \mathcal{S}}{\text{Fuel consumption by the offline optimal scheduling}} \quad (7)$$

We use the competitive ratio as a metric to evaluate the performance of a scheduling algorithm.

4. FROM A GENSET-ONLY TO A BATTERY-GENSET HYBRID SYSTEM

As discussed in Section 2, we want to find a strategy for an industry that already has a genset to use a battery to reduce its fuel consumption. We therefore make the assumption that the size of the genset, G, is given and it is large enough to meet the peak demands. We also assume that the genset can simultaneously charge the battery at its peak charging rate C. These assumptions are justified because of the following two reasons: (a) a genset that is sized for the peak is very likely to have excess capacity at most times, and (b) the marginal cost of increasing the size of a genset by a unit value decreases rapidly with genset size.

Given these assumptions, we never need to meet the load simultaneously from both the battery and the genset, hence, mode (iii) never arises. We now consider two problems: (a) the trade-off between battery size and carbon emission, (b) power scheduling between the genset and the battery (battery-genset power scheduling problem) to minimize total fuel consumption.

4.1 Battery-genset scheduling

As discussed in Section 2, the optimal scheduling algorithm for this problem must minimize the total time the genset is turned on. Batteries help to modify the load profiles such that the genset usage is minimized.

4.1.1 *Offline optimal scheduling*

Using Observation 1, the objective of the scheduling problem can be changed from minimizing total fuel consumption to minimizing the number of genset operating time slots. The offline optimal scheduling is therefore given by the following mixed-integer program:

Objective:

$$\min_{\mathcal{S}} \sum_{t=1}^{T} I_{t \in \mathcal{S}} \quad (8)$$

Subject To:

$$b(t) \leq b(t-1) - d(t) + GT_u I_g(t) \quad (9)$$

$$b(t) \leq b(t-1) + CT_u \quad (10)$$

$$0 \leq b(t) \leq B \quad (11)$$

where $I_{t \in \mathcal{S}}$ is one if $t \in \mathcal{S}$ and zero, otherwise. Constraints 9-11 ensure that the battery charging process in any mode obeys Eqs. (4)-(6).

The general battery-genset power scheduling problem is complex. Indeed, we prove it to be an NP-hard problem in general, by reducing a general instance of $0-1$ Knapsack problem. Before stating the theorem, we recap the Knapsack problem.

Knapsack problem: Given n objects of weights $w_1, w_2, \ldots, w_n$, a bag that can carry at most weight W, and a target value $P \leq n$, does there exist a subset of objects S such that $P \leq |S|$ and $\sum_{i \in S} w_i \leq W$?

The battery-genset scheduling problem can be stated as follows:
Battery-genset scheduling problem: Suppose that in the battery-genset system, the battery size is $B \geq 0$ with battery charging rate $C \geq 0$, and initial battery charge $B_0 \geq 0$. For the demands $d(1), d(2), \ldots, d(T)$ and a target value P, does there exist a genset schedule that turns on genset for at most $T - P$ time slots while ensuring that there is no loss of load?

THEOREM 1. *Battery-genset scheduling problem is NP- hard.*

PROOF. Let the battery size be $B = W$ and assume that it is fully charged at the beginning, i.e. $B_0 = W$. Let the battery charging rate be $C = 0$ and let the demands $d(t)$ for $1 \le t \le n$ be $d(t) = w_t$. For the same target value P, the genset scheduling problem returns yes iff the original Knapsack problem returns yes. Thus, we have reduced the Knapsack problem to a Battery-genset scheduling problem, showing that the later problem is at least as hard as the former problem. $\square$

4.1.2 Online near-optimal scheduling

In Section 4.1.1 we assumed that future load is given. In practice, however, prediction of stochastic demand is difficult. Here we study *alternate* scheduling, an online scheduling strategy, where no assumption is made regarding the future demand. It turns out that this scheduling is identical to the offline optimal scheduler if the battery charging rate permits fully charging the battery in a single time slot, as the following theorem demonstrates.

THEOREM 2. *If the battery charging rate is sufficient to charge the battery in a single time slot, i.e. $CT_u \ge B$, alternate scheduling is equivalent to the offline optimal schedule.*

PROOF. We prove the theorem by contradiction. Suppose that there exists an optimal offline scheduling different from alternate scheduling. We note that $CT_u \ge B$ implies that the battery is fully charged to its capacity B in a single time slot when the genset is on. Consider the first time slot t where the battery has sufficient charge to meet demand $d(t)$ but the optimal scheduling turns on the genset to meet the demand and charge the battery to B. Also, consider the earliest time slot $s > t$ when alternate scheduling turns on the genset because the battery is not sufficiently charged. Such a slot s exists as otherwise alternate scheduling will perform better than the optimal. After activation of genset in this slot s, the battery will be fully charged. Hence, at the end of slot s, alternate scheduling can be in a better state than the optimal to meet the future demand as the optimal will have a battery state of charge less than B. As both optimal and alternate have turned the genset on for the same number of time slots, we prove alternate scheduling is optimal. $\square$

We next compute the competitive ratio of the alternate scheduling. To simplify notation, we avoid ceiling and floor functions by assuming GT_u and B are divisible by CT_u. This assumption can be easily removed at the cost of additional notation.

THEOREM 3. *If the genset is large enough to simultaneously meet the peak demand and charge the battery at its maximum rate C, the competitive ratio for alternate scheduling is $\frac{k_1 \frac{G}{C} + k_2 T_u}{k_1 + k_2 T_u}$.*

PROOF. Consider the worst case demand profile $\mathcal{D} = \{d(1), d(2), \ldots, d(T)\}$ such that the battery is empty at the beginning of time slot 1 and at the end of time slot T for alternate scheduling. The battery is used in successive on and off cycles in the alternate scheduling as discussed before. Let m denote the total number of on/off cycles that the battery goes through during the T time slots. Our modeling assumptions enforce $CT_u \le GT_u \le B$ and $d(t) \le (G - C)T_u$ for all $t \in \{1, 2, \ldots, T\}$.

Correctness: For alternate scheduling, the battery is always charged by the genset at its peak rate C. Hence, the total number of charging time slots are exactly $\frac{mB}{CT_u}$. Note that the alternate scheduling discharges the battery to meet at least mB demand, which means $\sum_{t=1}^{n} d(t) \ge mB$. As the genset is the only source of energy and it can produce at most GT_u energy in a single time slot, the optimal scheduling will turn the genset on for at least $\frac{\sum_{t=1}^{n} d(t)}{GT_u} \ge \frac{mB}{GT_u}$ time slots. Using the definition of competitive ratio α, we get

$$\alpha \le \frac{k_1 G \frac{mB}{CT_u} + k_2 \sum_{t=1}^{T} d(t)}{k_1 G \frac{mB}{GT_u} + k_2 \sum_{t=1}^{T} d(t)} .$$

Observe that this ratio is maximum when the same additive term in the numerator and the denominator, $k_2 \sum_{t=1}^{T} d(t)$, achieves its minimum value (since $\alpha \ge 1$). Using $\sum_{t=1}^{T} d(t) \ge mB$, we get

$$\alpha \le \frac{k_1 G \frac{mB}{CT_u} + k_2 mB}{k_1 G \frac{mB}{GT_u} + k_2 mB} = \frac{k_1 \frac{G}{C} + k_2 T_u}{k_1 + k_2 T_u}$$

Tightness: The worst case is achieved for a periodic demand profile, where the demand is ϵ (which is non-zero but infinitesimally small) for the first $\frac{B}{CT_u}$ time slots and is $(G - C)T_u$ for the next $\frac{B}{(G-C)T_u}$ time slots and repeats the same format periodically. Let m be the number of periods in T time slots. In any period of the demand, alternate scheduling will turn on the genset for the first $\frac{B}{CT_u}$ time slots and discharge the battery for the remaining $\frac{B}{(G-C)T_u}$ time slots. The optimal scheduling will, however, always discharge the battery when the demand is ϵ. In each period of the demand being $(G - C)T_u$ for $\frac{B}{(G-C)T_u}$ time slots, the optimal turns the genset on to meet the demand and charges the battery for the first $\frac{B}{GT_u}$ time slots, followed by discharging the battery to meet the demand for the remaining $\frac{BC}{G(G-C)T_u}$ time slots. The ratio between fuel consumption by alternate and optimal is now given by

$$\alpha = \frac{k_1 Gm \frac{B}{CT_u} + k_2 mB}{k_1 Gm \frac{B}{GT_u} + k_2 mB} = \frac{k_1 \frac{G}{C} + k_2 T_u}{k_1 + k_2 T_u}$$

$\square$

In addition to these analytical observations, which suggest that the alternate scheduling performs quite efficiently, we will also numerically show that the alternate scheduling performs near-optimally for real demand traces from four commercial loads in Section 5.

4.2 Battery provisioning: cost/carbon footprint trade-off with alternate scheduling

This section computes the reduction in total fuel cost and total carbon emission achieved by adding a battery of size B to a genset-only system and using alternate scheduling. In absence of a battery, the cost of the system is given by the cost of fuel consumption (we ignore the genset purchase cost as a sunk cost). Here, total fuel consumption in time slots $[1, T]$ is the same as with genset-only scheduling, i.e.

$$\text{Fuel consumption} = k_1 GT + k_2 \sum_{t=1}^{T} d(t) .$$

We now estimate the number of time slots for which the genset is on after installation of a battery of size B and charging rate C and using alternate scheduling. Since the genset is large enough to charge the battery at its peak rate C, the genset is on for exactly $\frac{B}{CT_u}$ time slots in any battery charge-discharge cycle of alternate scheduling. Thus, to estimate the fraction of time the genset is turned off, we only need to calculate the expected number of slots, τ, in which the fully charged battery is discharged. The fuel consumption for the battery-genset hybrid system is then given by

$$\text{Fuel consumption} = k_1 GT \frac{\frac{B}{CT_u}}{\frac{B}{CT_u} + \tau} + k_2 \sum_{t=1}^{T} d(t) \qquad (12)$$

The life of a storage battery is reasonably well captured by the number of charge-discharge cycles. Suppose w.r.t. fuel cost, one battery charge-discharge cycle costs γ per kWh. We can estimate

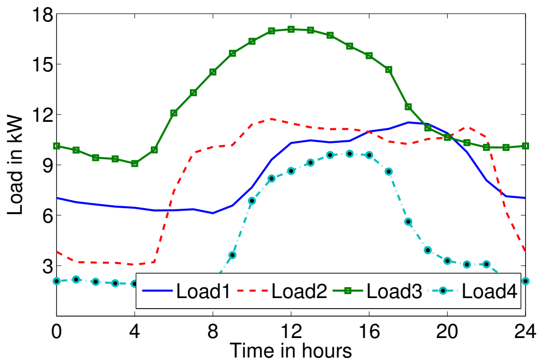

Figure 2: Average typical daily load profile for the four loads

the monetary expenditure as

$$\text{Cost} = k_1 G T \frac{\frac{B}{CT_u}}{\frac{B}{CT_u} + \tau} + k_2 \sum_{t=1}^{T} d(t) + \gamma \frac{T}{\frac{B}{CT_u} + \tau} \quad (13)$$

Eq. (12) and Eq. (13) allow us to compute the fuel consumption and investment cost of the battery-genset hybrid system for alternate scheduling as a function of the average duration of battery discharge τ.

Define the long-term average rate of the demand process as

$$\rho_d := E\left[\lim_{t\to\infty} \frac{\sum_{s=1}^{t} d(s)}{t}\right] \quad (14)$$

which is the ensemble average of the long-term average rates of all flows. A reasonable estimate for τ is the time needed to deplete a fully-charged battery with the long-term average rate. That is

$$\tau \approx \frac{B}{\rho_d} \quad (15)$$

• **Remark:** Eq. (15) in general holds as an approximation and not an exact description. To be more precise, it assumes that the expected total arrivals at any time interval is the long-term average rate multiplied by the size of the interval. This approximation however, becomes more accurate when the size of the time interval increases which is increasingly accurate when B increases. In other words, Eq. (15) becomes a more accurate description when B is not small.

Using the above approximation, we can further simplify Eq. (12) to obtain an insightful result that the fuel consumption is (roughly) independent of the battery size B and only depends on the battery charging rate.

$$\text{Fuel consumption} \approx \frac{k_1 G T}{1 + \frac{CT_u}{\rho_d}} + k_2 \sum_{t=1}^{T} d(t) \quad (16)$$

Note that this does not mean that the system improves genset efficiency even when the battery size is zero. First, the approximation in Eq. (15) loses its accuracy for small battery sizes. Moreover, there is an inherent lower bound of $CT_u \le B$ on the battery size.

5. EVALUATION

In this section we numerically evaluate our results for both scheduling and battery sizing of the hybrid battery-genset system by performing case studies using measured electricity consumption data from four commercial loads. The dataset, collected from a local distribution company in Ontario, Canada, contains hourly electricity consumption for four commercial loads for 18 months. We set time slot duration to be 5 minutes and linearly interpolate the demand for the 12 time slots in an hour[2]. Unless otherwise stated, we set $k_1 = 0.08415$ litre/kW and $k_2 = 0.246$ litre/kWh (i.e. $k_2/k_1 = 2.92$ 1/h) as given in [1]. To compute the offline optimal, we solve the mixed integer program described in Section 4.1.1 using Gurobi Optimizer.

Figure 2 shows the average daily load profile for the four commercial loads. Their mean demands were 8.53 kW, 8.59 kW, 12.85 kW, and 4.70 kW, respectively. The usage patterns in the graphs are due to variations in activities throughout the day.

5.1 Battery-genset scheduling

In Figure 3 and Figure 4 we examine the performance of the alternate scheduling with respect to the offline optimal scheduling. Figure 3 illustrates the total fuel consumption in 18 months for Load 1 using the alternate and offline optimal scheduling as a function of battery charging rate. Figure 4 also shows fuel consumption but as a function of the ratio of genset parameters k_2/k_1. We can make four important observations from these figures.

First, we note that for practical values of parameters, k_1 and k_2, carbon footprint is highly sensitive to the charging rate. Even for a small charging rate of 2 kW, which is 4 times less than the average load, total fuel consumption reduces by more than 10%. The gains can be more than 20% as the charging rate increases, however, this also increases the significance of our assumption that the genset is large enough to meet the demand and charge the battery.

Second, we note that alternate scheduling performs close to the offline optimal. Intuitively, this is true because real demands are stochastic and the worst cases rarely arise. Our worst case example in the proof of Theorem 3 has minimum demands tending to zero followed immediately by maximum demands $(G - C)T_u$. Such abrupt demand changes that are completely out of synch with the period of alternate scheduling are not common in practice.

Third, the graphs show that our analytical bound from Eq. (16) can closely estimate the total fuel consumption for alternate scheduling. This, in turn, indicates that our assumption in Eq. (15) can be accurate for practical loads.

Fourth, we see that savings decrease significantly as the value of genset affine function parameter k_2 increases. This is true because a large k_2 in Eq. (16) increases the weight of the term $\sum_{t=1}^{T} d(t)$, which remains unchanged with or without batteries.

5.2 Battery provisioning

Here we evaluate our observation from Section 4.2 that the total fuel consumption for alternate scheduling is almost insensitive to the size of battery if the battery is not very small. Figure 5 shows the total fuel consumption for Load 1 as the battery size increases for three different scheduling algorithms. It also shows the accuracy of our analytical bound from Eq. (16) in estimating the performance of alternate scheduling.

The graph verifies our claim that if the battery is greater than a small threshold, the total fuel consumption does not change as we increase the battery size. Specifically, this figure shows that for a carbon emission reduction target of 20%, one should buy a battery which has a capacity of at least 1 KWh and has a charging rate of 4 KW. In this example, the second constraint is not achievable by most Lithium-ion technologies (a standard '5C' 1 KWh has a nominal charging rate of only 0.2 KW). This suggests the use of an

[2]For $1 \le k \le 12$, kth time slot between successive hourly demands d_1 and d_2 will have an energy demand $d_1 + \frac{(d_2 - d_1)}{12}(k-1)$.

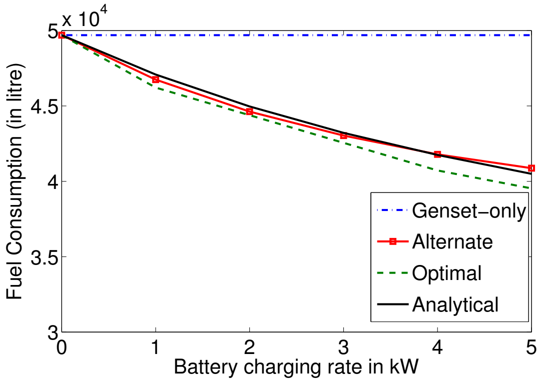

Figure 3: **Battery-genset scheduling:** Fuel Consumption vs battery charging rate for Load 1. $G = 25$kW, $B = 3$kWh, $k_1 = 0.08415$ litre/kW, and $k_2 = 0.246$ litre/kWh

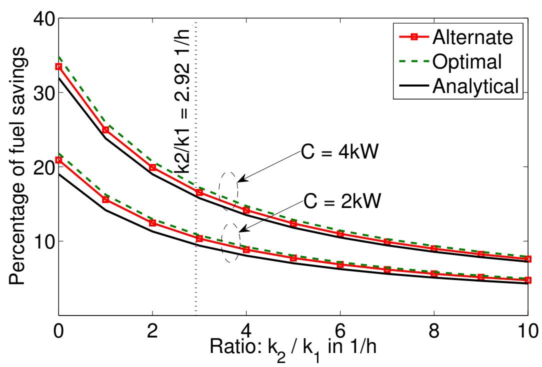

Figure 4: **Battery-genset scheduling:** Percentage of fuel savings vs ratio k_2/k_1 for Load 1. $B = 3$kWh and $G = 25$kW

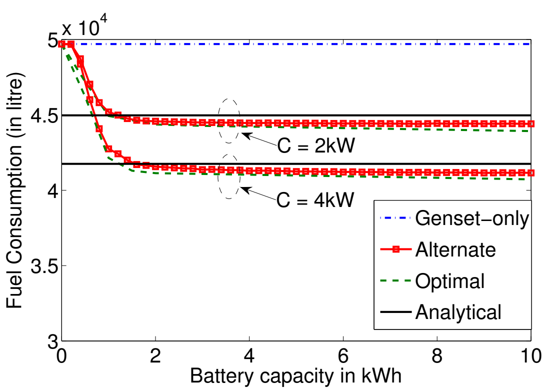

Figure 5: **Battery provisioning:** Fuel consumption vs battery capacity for Load 1. $k_1 = 0.08415$ litre/kW and $k_2 = 0.246$ litre/kWh

alternative technology, such as supercapacitors, which support fast charging, for such situations.

To ensure that this is not the artifact of some specific statistical properties of Load 1, we repeat the example for the other three loads in Figure 6. We observe the same behaviour for all the loads.

6. DISCUSSION AND CONCLUSION

Gensets are commonly used today to meet demands at places not connected to the central grid. Genset fuel efficiency is largest when

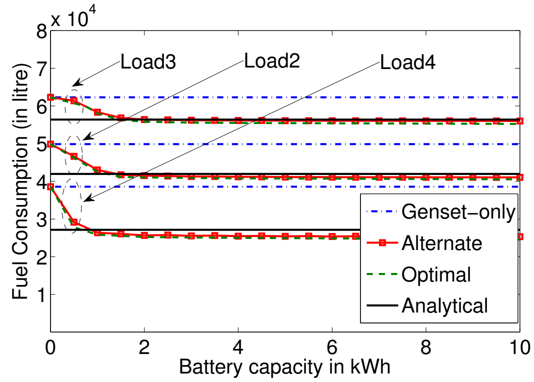

Figure 6: **Battery provisioning:** Fuel consumption vs battery capacity. $C = 4$kW, $k_1 = 0.08415$ litre/kW, and $k_2 = 0.246$ litre/kWh

it operates close to its capacity but due to a highly stochastic nature of demand, they are sized at the demand peak, and typically operate at a low efficiency of around 30% - 60%. A battery-genset hybrid system can be used to improve the genset efficiency. However, this involves designing a scheduling algorithm to allocate demand to be met by either the battery or the genset.

We address the problem of choosing a battery size to achieve a target carbon footprint reduction and that of scheduling a demand between the genset and the battery to minimize total fuel consumption. Although the general offline scheduling problem is NP-hard, we present a heuristic called *alternate* scheduling, an online scheduling strategy, that simply charges the battery to its fullest extent and then discharges it. We use demand traces to study the accuracy of our analysis and find that alternate scheduling performs close to the offline optimal. Using analytical and numerical techniques we present a counter-intuitive result that the total fuel consumption is not sensitive to the battery size, unless the battery is very small, when using alternate scheduling.

We realize that our work suffers from several limitations. We have assumed ideal batteries that are perfectly efficient and whose charging rate is independent of their size. We have assumed that the genset is large enough to simultaneously meet the load and also charge the battery. We have also only considered the tradeoff between battery size and carbon footprint for the case of alternate scheduling. Despite these limitations, we feel that our results are the first steps toward studying this important problem, and that our results provide a basis for additional work in this area.

7. REFERENCES

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