

# Distributed Control for Reducing Carbon Footprint in the Residential Sector

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**Abstract**—The current power grid is conservatively provisioned for rarely-occurring peaks. Expensive, quickly-ramping generators, typically with high carbon emissions, provide peak power. Therefore, it is possible to reduce both capital cost and carbon footprint by reducing the peak load. We address this issue by proposing to intelligently reduce loads from household appliances during peak times. Our scheme capitalizes on the fact that the power consumed by resistive loads can be reduced, at the cost of a small increase in appliance operation duration, with little impact on perceived user comfort. Specifically, on the receipt of congestion signals from the grid, appliance-based controllers intelligently reduce their load while ensuring that user comfort does not degrade below a pre-specified level. Simulations show that significant gains in energy reduction can be obtained with our scheme. For example, in Quebec, an estimated 12.9 MWh of peak power reduction can be obtained for a maximum of 10% increase in appliance operation duration.

## I. INTRODUCTION

Today’s power grid is conservatively designed to provide highly reliable service to consumers. The reliability is gained at the expense of extensive infrastructure in the form of generation, transmission and distribution systems [1]. These systems supply both a nearly-constant base load as well as a variable peak load. Systems that meet peak requirements are rarely used, yet add significantly to the capital cost, and, when used, introduce additional carbon footprint. Therefore, it is imperative to reduce the peak load to the extent possible.

One approach to peak-load reduction comes from exploiting the *elasticity* of household appliances [5]. An elastic component of an appliance, typically a resistor used for heating air or water, can instantaneously reduce its demand with no adverse impact on the appliance’s lifetime. With adequate insulation, when an elastic component reduces its demand by a factor  $\alpha$ , the time required to complete the operation of the appliance will be extended in inverse proportion to  $\alpha$ . However, the greater the increase in operation duration, the greater the user discomfort. Therefore, the factor by which the appliance duration is extended can be used as a measure of consumer comfort. Reference [5] quantified the potential in peak reductions from this approach. They were significant.

Our work builds on [5]. It embodies the following key insight: If information about the current state of the grid is supplied by the utility to the appliances in a periodic manner in the form of *congestion* signals, and if appliances operating elastic components have some delay budget remaining, they will be able to react to the signals from the grid so that the

congestion in the grid is eased. Congestion signals can reflect one of several different grid operating conditions, including lack of generation sources to meet aggregate peak demand or the excessive use of plants with high carbon footprint. These signals shift appliance use to demand slack periods without significantly affecting consumer discomfort.

Our work makes three main contributions:

- We design an appliance-side controller that responds to congestion signals from the grid to adjust the power consumption of elastic appliance components while causing only a bounded degree of user discomfort.
- We propose a design for an utility-side signal generator that adaptively computes congestion signals to be sent to appliances.
- We use simulations to demonstrate that our system reduces the peak load in response to congestion signals, as desired, with significant gains under realistic conditions. Moreover, the system is stable and responsive.

Our results indicate that if the systems proposed in this work were to be implemented in power grids today, the carbon footprint from generation sources commissioned to supply variable loads can be significantly reduced.

## II. RELATED WORK

Utilities have long used both price-based and incentive-based demand-management schemes for load shaping. These are extensively surveyed in Reference [6]). Because these schemes do not focus specifically on household appliances, we do not discuss them further.

This paper builds upon our prior work [5], where we introduced the notion of *elasticity* in appliance electricity consumption. Specifically, we studied the potential to carry out demand response by modifying ten common high-power appliances, such as dishwashers, baseboard heaters, and washing machines. Each appliance consists of several *components* whose distinctive periods of activity and inactivity during the operation of the appliance results in the appliance having a deterministic *nominal load profile*, that is, the power consumed by an unmodified appliance over the duration of its operation (see Figure 2 for an example). Some appliance components are elastic, that is, they can decrease their instantaneous power draw at the expense of increasing their duration of operation, but with no impact on the appliance’s lifetime. Assuming separate control of an appliance’s elastic components, we

quantified the relationship between the potential reduction in aggregate peak and the duration required to complete the operation of appliances in four geographic regions: Ontario, Quebec, France and India. We found that even with a small extension to the operation duration of appliances, peak demand can be significantly reduced in all four regions both during winter and summer.

Some appliances for sale today already include demand response features such as start time delay functions [4]. Appliances whose operation can be shifted (for example, dishwashers and washing machines) can potentially have their starting times be delayed according to pricing signals. Other appliances with thermal inertia (such as air conditioners and space heaters) can also potentially select a temperature setpoint according to pricing signals and consumer preferences. In this case, we can view the grid as “signalling” congestion state to appliances using pricing and appliances as responding to these signals by delaying their start times. This approach differs from ours in two ways. First, time of use pricing signals change rarely and do not reflect the instantaneous state of the grid. Second, postponing start times of appliances may result in noticeable discomfort to consumers.

Recent work has proposed scheduling frameworks for domestic appliances[7], [8], [9]. These schemes treat appliances as on-off loads. Moreover, these schemes do not quantify the decrease in user comfort due to their actions. In contrast, we examine properties of appliances at a finer granularity and ensure that the lifetime of appliances are not affected by varying power supplied to components that are sensitive to power fluctuations. We use realistic appliance statistics in the region that we have chosen to study based on our earlier work [5] (i.e., Quebec) to identify the gains in a set of homes, rather than in a single home.

### III. CONTROLLER DESIGN

#### A. Ideal controller

We first describe the characteristics of an ideal appliance controller:

- Every appliance has its own mode of operation: a dishwasher and a baseboard heater cannot be controlled in the same way. Therefore, the controller should take into account the idiosyncratic behaviour of each appliance.
- To reduce communication overheads, the controller should receive the least amount of information from the grid. In the limit, the controller should get a single signal that encodes the congestion state of the grid
- The controller’s behaviour should be decoupled from the reason *why* it was sent a congestion signal. This allows congestion signals to be generated from a variety of conditions, but all result in a decrease in the appliance’s peak load.
- The controller should result in a bounded degree of user discomfort. Specifically, once the duration of operation of the appliance has reached a pre-specified limit, the controller should ignore further congestion signals.

- The congestion signal should not directly dictate the power consumption of the appliance, because of the appliance’s idiosyncratic behaviour and the residual comfort budget. Instead, the appliance controller should make power adjustments independently based on inferred congestion trends.
- The controller should be stable, robust, and responsive.

The controller described next meets all of these requirements.

#### B. Fuzzy-logic based controller

Our appliance controller is designed as a fuzzy control system. This design decision is motivated by two considerations.

First, fuzzy logic can be used to represent and perform operations on elements that have partial belongingness to sets or labels. This maps well to the concept of grid overload. Specifically, with traditional “crisp” logic, the grid would be said to be *congested* when the demand exceeds a particular threshold and *uncongested* otherwise. Suppose that we consider the grid to be congested when the traffic load exceeds some threshold  $\theta$ . Then, it follows that a load of  $0.999\theta$ , which is very close to the congestion threshold, would still result in the grid being thought of being uncongested. It seems better, instead, to view each traffic load level to have different levels of belongingness to the variables ‘congested’ and ‘uncongested.’ This is the essential insight behind fuzzy logic.

The second motivation for our design decision is that the appliance controller needs to make a careful tradeoff between stability and responsiveness. A large body of prior work has shown that the use of fuzzy-logic based control results in the design of (non-linear) controllers that are robust, responsive, stable, and relatively insensitive to control parameters [11]. We therefore use fuzzy logic as the theoretical foundation of our work.

The design of fuzzy controllers is well understood [11]; therefore, we merely sketch the design of our system next. Note that in this description, we assume that time is discrete, with actions occurring only during discrete points in time.

#### C. Notation

The following is a list of the notation and functions used in our work. All notation is in the context of a particular appliance that is being controlled by a controller. Recall that an appliance consists of a set of elastic and inelastic *components* and that the appliance is in an elastic *phase* when any one of its elastic components is active.

- $\delta$ : The duration of a time step. This is typically 2 seconds to represent communication delay.
- $t_n$ : This is the time immediately after the  $n^{th}$  time step; that is,  $n$  time steps of  $\delta$  seconds have elapsed since some reference starting point.
- $S_c(t_n)$ : The signal that indicates the degree of congestion in the grid at time  $t_n$ .
- $B_o$ : This is the nominal duration of operation of the appliance (i.e., when it has not been modified).

- $\alpha$ : The fraction of time by which the appliance nominal operation duration  $B_o$  can be extended. We also refer to this as the appliance's *delay budget*.
- $\phi(j)$ : This function indicates whether phase  $j$  of an appliance's operation is elastic.
- $E_j(t_n)$ : This function returns the amount of energy still remaining to be consumed to complete phase  $j$  at time step  $t_n$ .
- $B_r(t_n)$ : This indicates the amount of delay budget remaining at time step  $t_n$ .
- $P_{min}^j$ : This is the minimum power that can be consumed by the appliance when it is in phase  $j$ . This will typically be set by the manufacturer.
- $P_n^j$ : This is the nominal power that is consumed by the appliance at phase  $j$ .
- $\Delta_n^j$ : This is the nominal time that is taken by the appliance to complete phase  $j$ .
- $P_m(t_n)$ : This is the minimum possible power the appliance can consume at time  $t_n$  so that budget requirements are met.  $P_{min}^j \leq P_m(t_n) \leq P_n^j$
- $P(t_n)$ : This is the actual power consumed by the appliance at time  $t_n$  due to the control action.

#### D. Updating Variables

The parameters presented above represent the state of the appliance at a given time. At every time step  $t_n$ , energy and budget parameters should be updated to reflect the appliance's current state. At every time step,  $E_j(t_n)$ , the amount of energy remaining to be consumed in phase  $j$ , is updated to  $E_j(t_{n-1}) - P(t_{n-1}) * \delta$ . Similarly,  $B_r(t_n)$ , the delay budget remaining, is updated to  $B_r(t_{n-1}) - \delta + \frac{\delta * P(t_{n-1})}{P_{n-1}^j}$ .

In order to meet the budget requirements, we introduced the term  $P_m(t_n)$ . This is computed as follows: Suppose the appliance is in phase  $j$  and  $\phi(j) = Elastic$  (no change to the nominal profile is made on a non-elastic phase) then

- If  $B_r(t_n) \geq \delta$  then  $P_m(t_n) = P_{min}^j$
- else  $P_m(t_n) = P_n^j$

Based on these updates, the appliance controller updates the actual power  $P(t_n)$  consumed according to the decision it makes based on the input congestion signal.

#### E. Controller Signal Input

The appliance controller receives from the grid a single input,  $S_c(t_n)$ , which reflects the congestion state of the grid at time  $t_n$ . The grid transmits a congestion signal to all the appliances every  $\delta$  seconds (typically 2 seconds) at time  $t_n$ . The appliance controller's interpretation of the congestion signal is independent of how  $S_c(t_n)$  is computed.

Three labels  $\{good, bad, worse\}$  are used to describe the congestion state of the grid. We define a membership function for each of these labels ( $\mu_g, \mu_b$  and  $\mu_w$ ) as illustrated in Figure 1. In these membership functions,  $x, y, T$  are parameters. We assume that  $x = y$  and  $T = 1 + x + z$  where  $z > 0$ . The last assumption is necessary as it makes no sense to define overlapping membership functions. As an

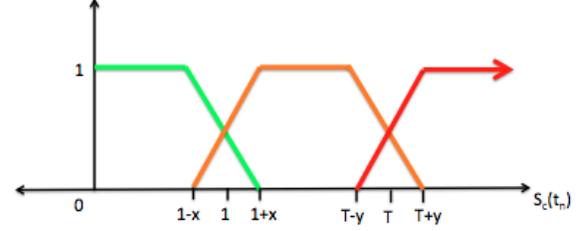


Fig. 1. Membership function for  $S_c(t_n)$

example of how the controller will interpret  $S_c(t_n)$ , suppose  $x = y = 0.1$  and  $S_c(t_n) = 0.95$  then the controller will use the membership functions to interpret the state of congestion of the grid to be 0.75 *good*, to be 0.25 *bad* and to be 0 *worse* (i.e., the grid is not in a worse condition). Hence, at  $t_n$ , the grid is neither completely in a *good* state nor is it in a completely *bad* state.

We next discuss how this fuzzified input will be used by the appliance controller for making control decisions.

#### F. Controller System

We propose the following fuzzy inferencing system for the appliance controller :

- R1: If congestion is *good* then increase power in *small* increments (additive increase)
- R2: If congestion is *bad* then decrease power in *small* decrements (additive decrease)
- R3: If congestion is *worse* then decrease power in *large* decrements (go to the minimum power possible)

This fuzzy inferencing system is composed of three rules each of which has one antecedent and one consequent. The antecedents in the rule set use linguistic labels presented in the membership function of Figure 1. The consequent in the rule set is one of the following three linguistic variables  $\{small\ increments, small\ decrements, large\ decrements\}$ . If at time  $t_n$ , the appliance is in phase  $j$  and  $\phi(j) = elastic$ , then the consequent of each rule is computed according to the following functions:

- *large decrements*  $P(t_n) = P_m(t_n)$ : the appliance should draw the minimum possible power while still meeting delay budget constraints.
- *small decrements*  $P(t_n) = \max[P_m(t_n), P(t_{n-1}) - \Delta w]$ : the appliance should either reduce its power consumption by  $\Delta w$  if the reduced power ( $P(t_n) = P(t_{n-1}) - \Delta w$ ) is greater than the minimum power or it should draw the minimum possible power otherwise
- *small increments*  $P(t_n) = \min[P_n^j, P(t_{n-1}) + \Delta w]$ : the component should either increase its power consumption by  $\Delta w$  if the increased power is lesser than the nominal power ( $P_n^j$ ) or it should take the nominal power otherwise

Once the consequent in each rule is computed, the controller's output, which is the actual value of  $P(t_n)$ , is com-

puted according to Equation 1.

$$P(t_n) = \frac{\mu_g(S_c(t_n)) \cdot C_1 + \mu_b(S_c(t_n)) \cdot C_2 + \mu_w(S_c(t_n)) \cdot C_1}{\mu_g + \mu_b + \mu_w} \quad (1)$$

#### IV. EVALUATION

In this section, we first consider a simplified illustrative example that demonstrates how a *single* appliance controller reacts to signals if it is used to control a dishwasher. We then study a more complex situation where we assume that appliance controllers are installed in 100 homes in Quebec. We have chosen Quebec based on our earlier results [5].

##### A. Examples of controller behaviour for a dishwasher

Recall that our controller responds to congestion signals no matter how they are generated. In this section, for simplicity, we consider a dishwasher with six phases of operation that receives a congestion signal whenever its load exceeds a value we call the *setpoint*, denoted  $S$ . The setpoint can be viewed, in this example, as the intrinsic amount of generation capacity available in, say, a microgrid, i.e., the system goes into overload whenever this setpoint is exceeded. The congestion signal, therefore, is defined by  $S_c(t_n) = \frac{P(t_n)}{S}$

The parameters of the controller are set to the following values:  $x = y = 0.1$ ,  $z = 0.2$ . There are two elastic phases, phases 2 and 4.  $P_{min}^j$  of the resistive components that are active during phases 2 and 4, are set to  $0.5 * P_n^j$ ,  $\alpha = 1.1$  and  $\Delta w = 5W$  respectively. In order to illustrate the impact of the signals on the behaviour of the appliance controller, three different values of  $S$  are considered and results are illustrated in Figures 2, 3 and 4.

For Figure 2 (resp. for Figure 3 and Figure 4),  $S = 1600W$  (resp.  $600W$  and  $1200W$ ).

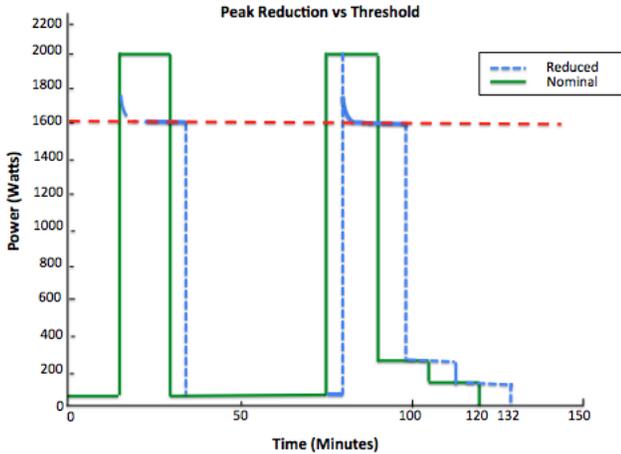


Fig. 2. Load profiles of dishwasher when  $S = 1600W$

In Figure 2, it is clear that since the setpoint is relatively high, the controller is able to adjust its power consumption so that the resistive heating element's power consumption rarely exceeds the setpoint of 1600 W. The appliance controller

is able to quickly react to the signals received and reduces appliance load to the setpoint value. Of course, this results in an extension of the duration of the elastic phase, but the overall delay budget is not exceeded.

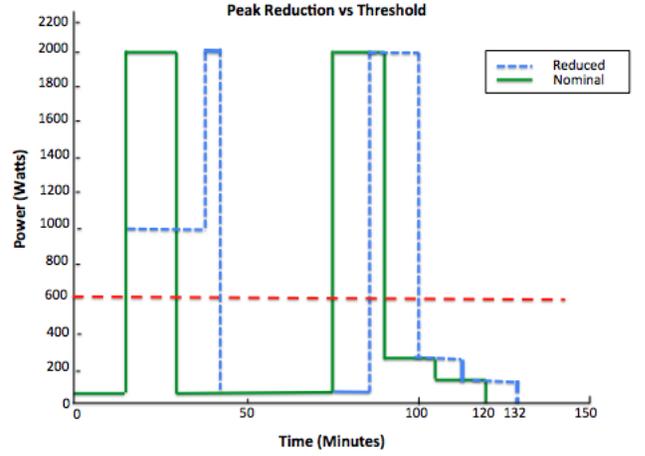


Fig. 3. Load profiles of dishwasher when  $S = 600W$

In Figure 3, the signal setpoint is set to a much lower value of 600W. Since  $P_{min}^j$  is set to 1000W for both elastic phases, the resistive heating element cannot consume power lower than 1000W. Once, the allocated time budget has depleted, the component returns to consuming its nominal power ( $P_n^j$ ) of that phase. We see that the appliance power increases to the maximum towards the end of the first elastic phase, reflecting the fact that the delay budget is used up. In the second elastic phase, there is no reduction in appliance power. This illustrates the fact that choosing a setpoint that is too low can result in a undesirable outcome.

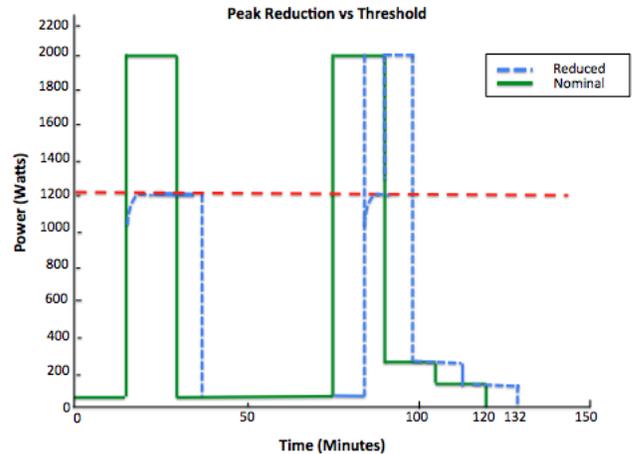


Fig. 4. Load profiles of dishwasher when  $S = 1200W$

In Figure 4, the signal setpoint is set to 1200W which is higher than  $P_{min}^j$  and lower than  $P_n^j$  for both elastic phases. When the power consumed by the dishwasher exceeds the signal setpoint, the controller reduces the power consumption

to  $P_{min}^2$ . The controller then enables the resistive heating element to find the setpoint in the first elastic phase. When phase 4, which is elastic, begins, the controller reacts in the same manner as in phase 2. However, after some time into the operation of phase 4, the time budget depletes. At this point the appliance controller returns the power consumption of the heating element to  $P_n^A$ .

Although the appliance controller operates independent of the way  $S_c(t_n)$  is computed, the three examples in this section show the importance of setpoint selection on the amount of energy that will be consumed above the signal setpoint. A setpoint that is too low can result in a perverse effect of no reduction in the peak load. The result of this effect on an ensemble of appliances is studied in our second example below.

### B. Large-scale study

In this section, we show results when using the appliance controller for a region of 100 homes in Quebec. We use Monte Carlo simulations for computing the results. Appliance properties used for the simulations are as in [5]. We assume that all appliance controllers use the *same* membership functions for interpreting the congestion degree of the grid as reflected by  $S_c(t_n)$ .

Our chosen performance metric is the reduction in peaking energy consumption, which we define as the reduction in aggregate energy consumption conditional on the aggregate appliance load exceeding the setpoint compared to the situation when appliances operate without the benefit of an appliance controller. This is the magnitude of the decrease in the amount of energy generated by high cost (or high carbon footprint) generators due to peaks in the appliance load as illustrated in Figure 5 (i.e., the value of the hatched area).

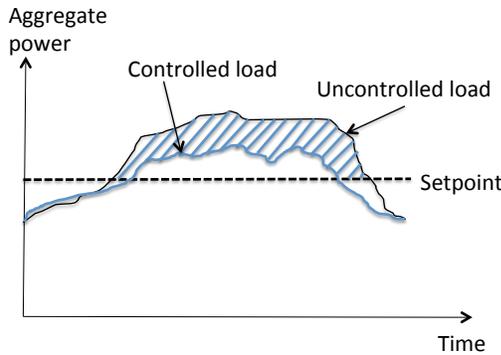


Fig. 5. Illustration of the performance metric

Signals are generated by the grid as  $S_c(t_n) = \frac{P_A(t_n)}{S}$  where  $P_A(t_n)$  is the aggregate power consumed at time  $t_n$  and  $S$  is the aggregate power setpoint, which corresponds to the crisp threshold after which plants with high carbon emissions are dispatched. Other parameters of the appliance controller are fixed, as before, to  $x = 0.1$ ,  $z = 0.2$ ,  $\Delta w = 5$  and  $P_{min} = \frac{1}{2}P_n^j$ .

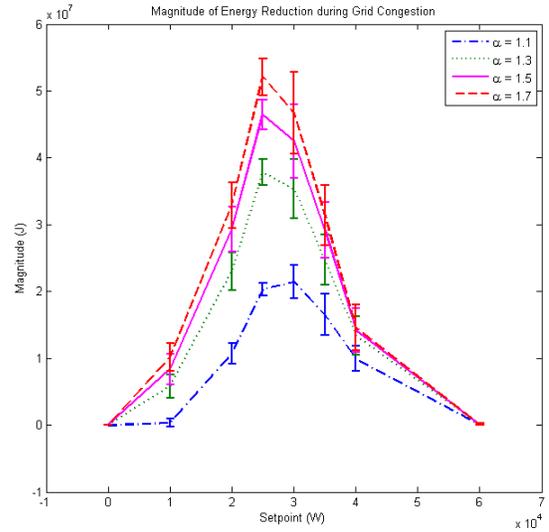


Fig. 6. Results when using static signalling for 100 homes in Quebec

The results of our simulations are shown in Figure 6 where the reduction in peaking energy consumption is shown as a function of the setpoint  $S$  for different values of  $\alpha$ . It is clear that there are significant reductions in peaking energy consumption during times of congestion for certain values of  $S$ . This is true for all four comfort factors (i.e., the four values of  $\alpha$ ). The greater the extension factor  $\alpha$ , the greater are the gains. We see that if the setpoint is high, no reduction in peaking energy is either required or achieved. Interestingly, when the signalling setpoint is set too low, there is no energy reduction at all! This is because when the setpoint is too low, the appliance controller reduces appliance power usage quickly to its minimum value, thus consuming the appliance delay budget very quickly. The controller subsequently ignores control signals so that it does not violate user comfort requirements. Consequently, the system behaves almost as if the controller were not operating at all. This unexpected behaviour is obviously not desirable: it is necessary to use an alternative mechanism for signal generation, as discussed next since the value of the setpoint cannot be chosen by the utility freely since it reflects the threshold after which energy becomes expensive.

### V. CONSISTENT SIGNAL GENERATION

In this section, we propose an adaptive mechanism for generating congestion signals that permits consistent reductions in peaking energy consumption, avoiding the unexpected behaviour presented earlier. Our scheme ensures that there is always a reduction in amount of peaking energy consumption independent of the choice of the setpoint. We do so by introducing a modified control signal that we call the *consistent congestion signal*  $S_c'(t_n)$ . Our modification is based on the insight that if lowering the setpoint causes a relative increase in peaking energy consumption then the setpoint should not

be lowered.

Specifically, in Figure 6 we found that when the setpoint was set to an approximate average of the aggregate power consumption, the gains were the greatest. Hence, in the proposed signal generator we use the moving average of the aggregate demand to compute the modified setpoint. We introduce a new variable denoted as  $ema(t_n)$  that represents an exponential moving average of the aggregate load.  $ema(t_n)$  is updated by the signal generator every  $\delta$  seconds according to Equation 2.

$$ema(t_n) = (1 - \beta) * ema(t_{n-1}) + \beta * P_a(t_n) \quad (2)$$

where  $\beta$  is a tuning factor and  $P_a(t_n)$  is the aggregate power consumption in the region at time  $t_n$ . At every timestep, the signal generator at the utility updates  $ema(t_n)$  and compares  $ema(t_n)$  with the chosen setpoint  $S$ . If  $ema(t_n) < S$ , then  $S'_c(t_n) = \frac{P_a(t_n)}{S}$  else  $S'_c = \frac{P_a(t_n)}{ema(t_n)}$ .

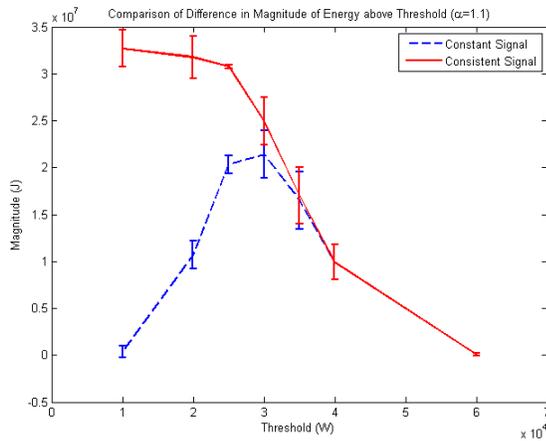


Fig. 7. Results for using adaptive signalling for 100 homes in Quebec

In Figure 7, we show the results obtained for  $\alpha = 1.1$  when the consistent congestion signal is used. The figure also shows the previously discussed signalling mechanism. Both systems use identical parameters. Additionally, for consistent signal generation, we choose  $\beta = \frac{1}{1800}$  which captures the intuition that the moving average should be computed over roughly one hour. It is clear from Figure 7 that, with consistent signal generation, even if a low setpoint is chosen, the gains from using our appliance controller are not lost. This makes it much easier for a system operator to choose a setpoint, in the firm knowledge that this will not result in a perverse outcome.

To understand what the gains could mean in reality, we next present the energy reductions in megawatt-hour (MWh) for Quebec. When the setpoint is set to 10,000W, the energy reduction gain for  $\alpha = 1.1$  is 35 MJ over a day as indicated in Figure 7 for 100 homes. In Quebec, there are 3.2 million households as of 2006 [12]. Hence, an average reduction of the energy consumed above the setpoint of 12.96 MWh can be expected in Quebec which is computed as follows  $\frac{3.5 * 10^7}{24 * 3600} * \frac{3.2 * 10^6}{100}$ . These results are very significant and are obtained for only a 10% increase in appliance operation duration, i.e., for

an appliance that operates for 50 minutes, this translates to an extension of only 5 minutes, which may not even be noticed by most consumers.

## VI. CONCLUSIONS

In this paper, we have presented a novel demand response mechanism that exploits appliance elasticity to decrease peak loads. We present the design of a fuzzy-logic based controller for appliances and a signal generator for the utility that can reduce the power consumed by appliances with elastic components that also have some delay budget remaining. We have shown that the proposed appliance controller is effective in being responsive to the state of the grid and allows for reduction in energy consumption during congested periods. Our proposed adaptive signal generator is able to result in significant energy reductions even for lower setpoints.

Our proposed demand response scheme allows loads to respond to grid congestion that arises either from excess demand or a shortfall in generation. This behaviour not only decreases the carbon footprint of the existing grid but will also ease integration of new distributed generation resources.

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